Bajaj College of Science, Wardha

(Formerly known as Jankidevi Bajaj College of Science)

(Autonomous)

Major Research Project

Title of the Project

Investigation of Space-Time Singularities in Gravitational Collapse of a Star

Principal Investigator

Dr. Sanjiv Shridharrao Zade

Associate Professor and Head,

Department of Mathematics

UGC Reference No. F.No. 41-797/2012 (SR)

Tenure of the Project : 01/07/2012 to 31/06/2015

Submitted to

University Grants Commission

Bahadur Shah Zafar Marg New Delhi – 110 002

Jamnalal Bajaj Marg, Civil Lines, Wardha – 442 001 (Maharashtra)

Website : www.jbsw.shikshamandal.org

Email : jbsciencewardha@yahoo.co.in

CONTENTS

Item No	Item	Details
[1]	Title of the Major Research Project:	**Investigation of Space-time Singularities in Gravitational Collapse of a Star**
[2]	Name and address of the Principal Investigator:	**Dr. Sanjiv Shridharrao Zade** **Dept. of Mathematics, J.B.College of Science, Wardha (M.S.) PIN- 442001**
[3]	Name and address of the instiuion:	**Jankidevi Bajaj college of Science, Wardha. Jamnalal Bajaj Marg, Civil lines, Wardha (M.S) PIN- 442001**
[4]	UGC approval Letter No. and Date:	**F. 41-797/2012 (SR) Dated 18 July 2012**
[5]	Date of Implementation:	**01.07.2012**
[6]	Tenure of the project:	**01.07.2012 to 30. 06.2015**
[7]	Grant Allocated:	**Rs. 2,58,000/-**
[8]	Total Grant Received:	**Rs. 2,05,500/-**
[9]	Final Expenditure:	**Rs. 1,85,257/-**

[10] Title of the Project: Investigation of Space-time Singularities in Gravitational Collapse of a Star

[11] Objectives of Project:

A sufficiently massive star (heavier than 5 solar masses) which has exhausted all its nuclear fuel could undergo an endless gravitational collapse without attaining any equilibrium state. The state of ongoing collapse proceeds to form a **space-time singularity**.

The **objectives** of the project are:

1) To investigate the possibility of cosmic censorship violation in gravitational collapse of various spherically symmetric and non-spherically symmetric space-times by finding counter examples to it.
2) To study the gravitational collapse of the neutral and charged quark matter.
3) To generalize the earlier studies of the spherically symmetric gravitational collapse to higher-dimensions.
4) To analyze the strength of singularity using the Clarke and Krolak criteria.
5) To find a simpler way to solve the differential equation of radial null geodesic.
6) To use computer for calculations for accurate and error-free results.

[12] Whether objectives were achieved: Yes. The proposed objectives of the project were achieved.

[13] Achievements from the Project:

As proposed in the objectives of this project, gravitational collapse of various space-times has been studied. The earlier studies of the spherically symmetric gravitational collapse were successfully generalized to higher-dimensions. The existence of singularities was determined using a simpler method. In this method the differential equation of radial null geodesics was converted to a linear equation which can be solved easily. The existence of positive roots determined the visibility of singularity. Violation of cosmic censorship hypothesis has been shown in our paper.

Strength of singularity was determined using the Clarke and Krolak criteria. Three research papers on the work have been published in journals.

[14] Summary of the Findings

A space-time singularity is a region of the space-time where the quantities like energy density and curvature scalar diverge and all the laws of physics break down. Singularities could occur in cosmology at the origin of the universe or in the continued gravitational collapse of a massive star at the end of its life cycle. The Cosmic Censorship Hypothesis (CCH) proposed by Roger Penrose states that the singularities formed in the generic gravitational collapse of a star are always hidden behind the event horizon of gravity. The CCH implies that the end state of gravitational collapse of a massive star is always a black hole which conceals the resulting space-time singularity. There is considerable interest in the CCH to test its validity, but neither a general proof nor any mathematical formulation is available so far. Thus to prove or to find a counter example to this hypothesis is one of the most important areas of mathematical research today.

In this project, we have studied the gravitational collapse of stars; investigated the visibility of space-time singularities and examined the possibility of cosmic censorship violation in gravitational collapse some space-times. The earlier studies of the spherically symmetric gravitational collapse have been generalized to higher-dimensions. The results of occurrence of naked singularity in higher dimensions are important as they indicate the possibility of existence of extra dimensions in the universe.

(i) In the published paper 'Dynamics of Apparent Horizons in (N+2)-Dimensional Space-Times' (Paper-1, Appendix -II), the formation of central singularities in (N+2) dimensional Tolman-Bondi space-time has been studied. It has been shown that the time of formation of central singularity decreases with the increase in the dimension of space-time. For $N \geq 4$, the apparent horizon forms earlier than the central singularity hence the singularity will be a black hole. Thus CCH is respected for spacetimes having $N \geq 4$. Thus the end state of gravitational collapse is a black hole.

(ii) We have studied the gravitational collapse of higher-dimensional monopole-Vaidya solution. We have shown that naked singularities do occur as the end stage of gravitational collapse in $N + 2$ dimensional monopole-Vaidya solution violating the CCH. It has been shown that results in 4-dimensional space time are also valid in $(N + 2)$ dimensional space-times. Thus the conclusion that the dimension of the space-time does not play any fundamental role in the formation of the naked singularities (Ref. Paper-2, Appendix -II)

(iii) We have analyzed the structure of the space-time singularities formed in the gravitational collapse of Monopole-Radiating dyon solution in Anti-de Sitter space-time. It is shown that the singularities are not hidden behind the event horizon. It is revealed that the final outcome of the collapse depends on the electric and magnetic charge parameters. The strength of the singularity has been studied and it has been shown that these are gravitationally strong (Ref. Paper-3, Appendix -II).

(iv) In our work, a simpler method has been used to investigate the nature of singularities. The differential equation of radial null geodesics emanating from the singularity is transformed to an algebraic equation. The occurrence of positive roots indicates a naked singularity. The absence of positive roots indicates a black hole as the end state of gravitational collapse. We have used a computer program to solve the equations. (Ref. Papers-1, 2, 3).

[15]_ Contribution to the Society:

The research work carried out in his MRP have indirect societal benefits. It contributes to the knowledge of in the area of gravitational collapse, naked singularities of astrophysics. It enhances the understanding of space exploration and the universe. The study is useful to researchers in this area as it gives insight into fundamentals and advancements. It may not directly impact the immediate daily life but would foster the scientific progress in long term

[16] Whether any Ph.D. enrolled/produced out of the project: No

[17] No. of Publications out of the project: 03

Following **three** papers have been published in International **journals** (Re-prints attached)

1. Dynamics of Apparent Horizons in (N+2)-Dimensional Space-Times *Chinese Physics Letters* Vol.**29** (11) 140401 (2012). ISSN : 0256-307X

2. Final Fate of (n+2)-Dimensional Monopole Vaidya Solution, *International Journal of Mathematical Research & Science (IJMRS)* Vol. 1 Issue 5, October – 2013. ISSN: 2347 – 3975

3. Monopole-Radiating Dyon Solution in Anti-De Sitter Space Time, *International Journal of Engineering Research & Technology (IJERT)* Vol. 2 Issue 11, November – 2013. ISSN: 2278-0181

Following **two** papers have been presented in National/International conferences (Certificates attached)

1. Effect of monopole field on the gravitational collapse of (n+2)-dimensional Vaidya space-time, National Conference on Statistics, Mathematics and their applications, Institute of Science, Nagpur, January 3-4, 2014.
2. Final fate of (n+2)-dimensional Monopole Vaidya Solution, International Conference on recent Advances in Mathematics, RTM Nagpur University, Nagpur, January 20-23, 2014.

(Principal Investigator)

UNIVERSITY GRANTS COMMISSION
BAHADURSHAH ZAFAR MARG
NEW DELHI-110 002

UTILIZATION CERTIFICATE

It is certified that as against grant of Rs. 2,58,000/- (Rupees Two Lakh Fifty Eight Thousand only) sanctioned to Jankidevi Bajaj College of Science, Wardha by University Grants Commission vide letter No. **F. No. 41-797/ 2012 (SR) dated 18th July 2012** and released of Rs. 2,05,500/- (Rupees Two Lakh Five Thousand Five Hundred only) towards Major Research Project entitled **"Investigation of space-time singularities in Gravitational of a Star"**, undertaken by **Prof. Dr. Sanjeev S. Zade,** an amount of Rs. 1,85,257/- (One Lakh Eighty Five Thousand Two Hundred Fifty Seven only) has been spent for the purpose for which it has been sanctioned in accordance with the terms and conditions laid down by the University Grants Commission.

If as a result of check or audit objection, some irregularity is noticed at a later stage action will be taken to refund or regularize the objected amount.

Signature of the Principal,
Jankidevi Bajaj
College of Science, WARDHA

Signature of Principal Investigator

For TAPDIYA CHANDNA BHUTADA & CO.
CHARTERED ACCOUNTANTS
(RAJENDRA BHUTADA - PARTNER)
Membership No. 43283
Signature of the Chartered Accountant
with Seal & Regd. No. of C.A.

UNIVERSITY GRANTS COMMISSION
BAHADURSHAH ZAFAR MARG
NEW DELHI-110 002

STATEMENT OF EXPENDITURE IN RESPECT OF MAJOR RESEARCH PROJECT

1.	Name of the Principal Investigator	:	**Dr. Sanjeev S. Zade**
2.	Deptt. of University / College	:	**Jankidevi Bajaj College of Science, Wardha-442001 (M.S.)**
3.	UGC approval No. and Date	:	**F. No. 41-797/2012 (SR) dated 18th July 2012.**
4.	Title of the Project	:	**"Investigation of space-time singularities in Gravitational of a Star".**
5.	Effective date of starting the Project	:	**01st July 2012**
6.	Period of Expenditure	:	**01st July, 2012 To 30th June, 2015**
7.	Details of Expenditure	:	

Sr. No.	Item	Amount Approved. Rs.	Grant Received Rs.	Expend. incurred Rs. 2012-13	Expend. incurred Rs. 2013-14	Expend. incurred Rs. 2014-15	Expend. incurred Rs. upto 30-06-15	Total Expen-diture
A.	**Non-Recurring**							
1	Books and Journals	50,000	**1,50,000**	-	16,598	23,789	-	**40,387**
2.	Equipments	1,00,000		67,690	33,985	-	-	**1,01,675**
B.	**Recurring**							
3.	Contingency	30,000	**55,500**	-	12,055	9,325	8,940	**30,320**
4.	Travel / Field Work	75,000		-	-	8,155	4,720	**12,875**
5.	Chemicals& Glassware	0		-	-	-	-	-
6.	Hiring Services	0		-	-	-	-	-
7.	Special Need	0		-	-	-	-	-
8	Overhead Charges @ Rs.10% approved recurring Grant (Except Travel & Field Work)	3,000		-	-	-	-	-
	Total	**2,58,000**	**2,05,500**	**67,690**	**62,638**	**41,269**	**13,660**	**1,85,257**

For TAPDIYA CHANDNA BHUTADA & CO.
CHARTERED ACCOUNTANTS

(RAJENDRA BHUTADA - PARTNER)
Membership No. 43283

Signature of the Principal
Principal
Jankidevi Bajaj
College of Science, WARDHA

Signature of Principal Investigator

Signature of the Chartered Accountant with Seal & Regd. No. of C.A.

Annexure - IV

UNIVERSITY GRANTS COMMISSION

Bahadur Shah Zafar Marg, New Delhi – 110 002

STATEMENT OF EXPENDITURE INCURRED ON FIELD WORK

Name of the Principal Investigator: Dr. Sanjiv S. Zade

Name of the place visited	Duration of the Visit		Mode of Journey	Expenditure Incurred (Rs.)
	From	To		
Chennai Mathematical Society (CMI), Chennai	14.3.2015	18.3.2015	Airplane & Railway	8,155
Jadavpur University Calcutta, National Library Calcutta Calcutta Mathematical Society,	4.6.2015	7.6.2015	Airplane & Railway	4,720

Grand Total: 12,875/-(In words: Twelve thousand eight hundred seventy five only)

Certified that the above expenditure is in accordance with the UGC norms for Major Research Projects.

(Dr. S. S. Zade)
Principal investigator

Principal
Principal
Bajaj College of Science
Wardha

विश्वविद्यालय अनुदान आयोग
University Grants Commission
(मानव संसाधन विकास मंत्रालय, भारत सरकार)
Ministry of Human Resource Development, Govt. of India)
बहादुर शाह जफर मार्ग, नई दिल्ली – 110 002
Bahadur Shah Zafar Marg, New Delhi – 110 002

ज्ञान-विज्ञान विमुक्तये

सत्यमेव जयते

UGC Website: www.ugc.ac.in
Ph. 011-23604414 (CPP-I/Colleges)

Speed Post

F. No. 1-1/2018(CPP-I/C)

July, 2018

The Principal,
Jankidevi Bajaj College of Science
Wardha, Jamnalal Bajaj Marg,
Civil Line, Wardha – 442 001
Maharashtra

Sub: - Recognition of college under Section 2(f) and 12(B) of the UGC Act, 1956.

Sir,

With reference to your letter No. J.B.C./174/2018-2019 dated 15-06-2018 on the above subject, I am directed to say that the name of **Jankidevi Bajaj College of Science, Jamnalal Bajaj Marg, Civil Lines, Wardha – 442 001, Maharashtra** established in the year of **1962** affiliated to **Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur** has already been included in the list of Colleges maintained under Section **2(f) and 12(B)** of the UGC Act, 1956 under the head **Non-Government** College teaching upto **Master's** Degree.

Yours faithfully,

(Pramod Sharma)
Under Secretary

No. DEV/AC/G/347 **2 F**

From

The Development Officer,
Nagpur University,
Nagpur

To,

The Secretary,
University Grants Commission,
Bahadur Shah Zafar Marg,
NEW DELHI-1.

Dated Nagpur, the 22nd August, 1975

Subject:- Submission of Indemnity Bonds by the Colleges falling under clause 1(IV)-Recognition of Colleges under Section 2(f)

Reference:- Your letter No. F. 33-34/65 (CD/CP) Pt. V. Dated 1-7-1975
and No. DEV/AC/G/347
This office letters
1) No. Dev/AC/G/163, dated 29-6-1975,
2) No. DEV/AC/G/187, dated 30-6-1975.

Sir,

I am directed to refer to your letter under reference on the above subject and to forward herewith Indemnity Bonds on Stamp paper and copy of resolution in respect of 21 colleges (as per list enclosed) temporary affiliated to this University for further necessary action. The above mentioned colleges are already included in the approved list of colleges under Section 2(f) of the U.G.C. Act, 1956.

Encl: As above

Yours faithfully,
[signature]
Development Officer.

NAGPUR UNIVERSITY

No. DEV/AC/ F/2603 Dated Nagpur, the 22nd August, 1975.

Copy forwarded to the Principal ______ ______ for information.

[signature]
Development Officer,
Nagpur University.

VBN/

10

108
22-9-98

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI - 110002.

No.F. 8-24/97 (CPP-I) September 1998

11 SEP 1998

The Rgistrar
Nagpur University,
Nagpur-440010.

Sub:- Declaring a college fit to receive assistance under Section 12-B of the UGC Act, 1956.

Sir,

I am directed to refer to your letter No. JBC/3112/98-99 dated 19/7/98 on the above subject and to say that it has been noted that the Jankidevi Bajaj College of Science, Wardha has been granted permanent affiliation by the Nagpur University. Accordingly, Jankidevi Bajaj College of Science, Wardha which already stands included in the list of colleges maintained under Section 2(f) of the UGC Act, 1956 is declared fit to receive assistance from the UGC, and other central sources in terms of Rules framed under Section 12-B of the UGC Act, 1956.

Yours faithfully,

(D.D. Mehta)
Under Secretary

Copy to:-

1. The Principal, Jankidevi Bajaj College of Science Wardha-442001.
2. The Secretary, Govt. of India, Ministry of Human Resource Development (Deptt. of Education) New Delhi.
3. Joint Secretary, UGC Office, Industrial Chemical Laboratory, Near Poona University Campus, Pune-411007.
4. All Sections in the UGC.
5. Section Officer (FD-III Section), UGC.
6. DTP Cell, UGC.
7. Guard file.

(C.P. Arora)
Section Officer

1) original in Affiliation File
2) Xerox in Master File
3) Copy to Shikshan Mandal for information

22/9/98

Dynamics of Apparent Horizons in (N+2)-Dimensional Space-Times

S. S. Zade

Department of Mathematics, J. B. College of Science, Wardha, India

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Abstract

We analyze the time of formation of central singularities in $(N+2)$-dimensional space-times. We show that the time of formation of central singularity decreases with the increase in dimensions of the space-time. We have studied the dynamics of trapped surfaces formation in different higher dimensional space-times and compared them graphically.

Keywords: Gravitational collapse; naked singularity; black hole.

PACS: 04.20.Dw, 04.20.Cv, 04.70.Bw

In the framework of Einstein theory of relativity space-time singularities can be formed by a gravitational collapse. The examples of naked singularities known in gravitational collapse arise from various forms of matters. These include dust,[1,2] radiation[3,4] and null strange quark fluid.[5,6] Solutions of the Einstein equations in higher dimensions have come to play an important role in general relativity. Recently there has been a great deal of interest in models wherein the size of the extra dimensions is much larger than the Planck length.[7,8] It is now important to consider the evolution of extra dimensions, since the observed strength of the gravitational force is directly proportioned to the size of the extra dimensions (c.f. 9).

The CCH implies that the black holes are generic final states of the gravitational collapse. In the present work, we would like to see whether the large extra dimension can throw some light on the validity of CCH. Does a gravitational collapse always lead to the formation of a naked singularity? What kind of singularities can exist in nature? Can these singularities, if they are naked, carry over to higher-dimensional space-times? These are among the most important problems in general relativity and have been studied for a long time. It has been argued in Ref. [7] that multi-dimensional space-times where all dimensions are considered on an equal footing, are not so realistic, as we are living in an effectively four-dimensional space-time, so in principle one might expect that by dimensional reduction the multi-dimensional space times should reduce to our four-dimensional world.

To investigate study the dynamics of the formation of apparent horizons and the time of formation of central singularity in different dimensions, we study the $(N+2)$-dimensional spherically symmetric Tolman-Bondi space-time given by the line element[10,11]

$$ds^2 = -dt^2 + \frac{R'^2}{1+f(r)}dr^2 + R^2 d\Omega^2, \tag{1}$$

where

$$\begin{aligned} d\Omega^2 = d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3^2 + \cdots \\ + \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta_3 \cdots \sin^2\theta_{N-1}\, d\theta_N^2, \end{aligned} \tag{2}$$

is the metric on N-sphere.

The energy momentum tensor is given by

$$T^{ab} = \epsilon(t,r)\, \delta_a^t\, \delta_b^t, \tag{3}$$

where $\epsilon(t,r)$ is known as energy density, and is given by

$$\epsilon(t,r) = \frac{NF'}{2R^N R'}. \tag{4}$$

$F(r)$ and $f(r)$ are arbitrary functions of r; known as mass function and energy function respectively. The area radius R is a function of radial co-ordinates r and t and satisfies the following equation

$$\dot{R}^2 = \frac{F(r)}{R^{N-1}} + f(r). \tag{5}$$

(We have set up $8\pi G/c^4 = 1$).

For simplicity, we consider marginally bound collapse i.e. $f(r) = 0$. Since for collapsing case we require $\dot{R} < 0$.

Eq. (5) gives

$$\dot{R} = -\frac{\sqrt{F}}{R^{(N-1)/2}}, \tag{6}$$

integrating which, we get

$$R^{(n+1)/2} = \left[r^{(N+1)/2} - \left(\frac{N+1}{2}\right)\sqrt{F}\,t\right], \tag{7}$$

where we have used the freedom in scaling of the comoving coordinate r to set up $R(0,r) = r$ at the starting epoch of collapse.

If we consider $t_c(r)$, the time at which area radius R becomes zero, then from Eq. (7) we get

$$t_c(r) = \left(\frac{2}{N+1}\right)\frac{r^{(N+1)/2}}{\sqrt{F}}. \tag{8}$$

Here the ranges of r and t are:

$$0 \leq r < \infty, \ -\infty < t < t_c(r). \tag{9}$$

It follows from Eq. (6) that the apparent horizon for the (N+2)-dimensional dust cloud is given by

$$R\left[t_{ah}(r), r\right] = F^{1/(n-1)}, \tag{10}$$

where $t_{ah}(r)$ is the time at which apparent horizon forms.

If the neighborhood of the center gets trapped earlier than the singularity, then the central singularity could be naked with radial null geodesics escaping from it.

Using R from Eq. (10) into Eq. (7), we obtain

$$t_{ah}(r) = \left(\frac{2}{N+1}\right)\frac{r^{(N+1)/2}}{\sqrt{F}} - \left(\frac{2}{N+1}\right)F^{1/(n-1)}. \tag{11}$$

Inserting the expression for $t_c(r)$ from Eq. (8) into above equation, we get

$$t_{ah}(r) = t_c(r) - \left(\frac{2}{N+1}\right)F^{1/(n-1)}. \tag{12}$$

Since $F(r)$ is strictly positive for $r > 0$, it follows from the above equation that

$$t_{ah}(r) < t_c(r) \text{ for } r > 0.$$

It means apparent horizon will form before the formation of the singularity, hence the singularity will not be visible in this case, leading the collapse to a black hole.

But at $r = 0$, $t_{ah}(0) = t_c(0)$. So there is a chance for some radial null geodesics to escape from the apparent horizon. Thus it is the central singularity point $r = 0$, whose nature, in terms of being naked or covered, depends upon the initial data.

Now we define

$$\epsilon(0, r) = \rho(r).$$

Hence from Eq. (4), we obtain

$$F(r) = \frac{2}{N} \int \rho(r)\, r^N dr. \tag{13}$$

We assume $\rho(r)$ to be Taylor expandable and decreasing away from the center. So $\rho(r)$ will be of the form [12]

$$\rho(r) = \rho_0 + \rho_1 r + \frac{\rho_2 r^2}{2!} + \frac{\rho_3 r^3}{3!} + \cdots + \frac{\rho_n r^n}{n!} + \cdots . \tag{14}$$

In the above equation, as the density is decreasing away from the center, the first non-vanishing derivative of the density is negative. We then have

$$F(r) = F_0 r^{N+1} + F_1 r^{N+2} + F_2 r^{N+3} + \cdots , \tag{15}$$

where

$$F_n = \frac{2}{N} \frac{\rho_n}{n!(n+1+n)}, \quad n = 0, 1, 2, \cdots . \tag{16}$$

We consider those density functions which decrease as we move away from the center, hence the first non-vanishing derivative of the density at the center is negative. Therefore we consider the expression for $F(r)$ as

$$F(r) = F_0 r^{N+1} + F_n\, r^{N+1+n}.$$

Inserting the above expression for $F(r)$ and using binomial expansion, we obtain the equation

$$t_{ah}(r) = t_c(0) - \left(\frac{1}{N+1}\right) \frac{F_n}{F_0^{3/2}}\, r^n - \left(\frac{2}{N+1}\right) F_0^{\left(\frac{1}{N-1}\right)}\, r^{\left(\frac{N+1}{N-1}\right)}, \tag{17}$$

where $t_c(0)$ is the time of occurrence of the singularity at the center $r = 0$ (i.e. central singularity). By putting $r = 0$ and inserting the expression for $F(r)$ in Eq. (8) we get $t_c(0) = \frac{2}{(N+1)\sqrt{F_0}}$.

Eq. (17) is the general formula, which gives the time of formation of apparent horizon in (N+2)-dimensional space-times.

At a general point, the behaviour of the apparent horizon will depend on the nature of $F(r)$, but at the center, i.e. $r = 0$, it is determined by the first non-vanishing derivative of density. In Eq. (17), F_n is associated with the first non-vanishing derivative of density ρ_n through Eq. (16), hence it has negative value. We explore Eq. (17) for different values of N.

Case 1: $N = 2$.

For $N = 2$, Eq. (17) reduces to the four-dimensional space-time. It has been shown in Ref. [13] that if the first non-vanishing derivative of density at the center is either ρ_1 or ρ_2 then $t_c(0) < t_{ah}(r)$ always holds. But if the first non-vanishing derivative of density at the center is ρ_3, then

$$t_c(0) < t_{ah}(r)$$

if

$$\xi = \frac{F_3}{F_0^{5/2}} < -2. \tag{18}$$

Case 2: $N = 3$.

For $N = 3$, the expression (17) reduces to the five-dimensional space-time. In this case

(i) If the first non-vanishing derivative of density at the center is ρ_1, then to the leading order, Eq. (17) becomes

$$t_{ah}(r) = t_c(0) - \frac{F_1}{4F_0^{3/2}}\, r. \tag{19}$$

Since $F_1 < 0$, the term $-\left(\frac{F_1}{4F_0^{3/2}}\right) r$ will be positive, hence $t_c(0)$ would be less than $t_{ah}(r)$, which means center will get trapped before its neighboring region. Therefore, in this case, the singularity is naked.

(ii) If the first non-vanishing derivative of density is ρ_2, i.e. $n = 2$, then Eq. (17) becomes

$$t_{ah}(r) = t_c(0) - \left(\frac{F_2}{4F_0^{3/2}} + \frac{\sqrt{F_0}}{2}\right) r^2. \tag{20}$$

From which it is clear that

$$t_c(0) < t_{ah}(r)$$

if

$$\frac{F_2}{F_0^2} < -2. \tag{21}$$

The above result is in agreement with the result in Ref. [10]

(iii) If the first non-vanishing derivative of density is ρ_3, then Eq. (17) gives

$$t_{ah}(r) = t_c(0) - \frac{\sqrt{F_0}}{2} r^2, \tag{22}$$

which clearly suggests that $t_{ah}(r) < t_c(0)$, leading the collapse to a black hole.

Case 3: $N \geq 4$ (i.e. $D \geq 6$).

(i) If the first non-vanishing derivative of density is ρ_1, then Eq. (17) becomes

$$t_{ah}(r) = t_c(0) - \frac{F_1}{(N+1)F_0^{3/2}} r - \left(\frac{2}{N+1}\right) F_0^{\left(\frac{1}{N-1}\right)} r^{\left(\frac{N+1}{N-1}\right)}. \tag{23}$$

In this case, we note that

$$1 < \frac{N+1}{N-1} < 2.$$

But if we consider N very large, then $(N+1)/(N-1)$ nearly equals 1. Hence combining the second and third term in Eq. (23), we get

$$t_{ah}(r) = t_c(0) - \frac{1}{N+1}\left[\frac{F_1}{F_0^{3/2}} + 2F_0^{\left(\frac{1}{N-1}\right)}\right] r. \tag{24}$$

From above expression it is clear that

$$t_c(0) < t_{ah}(r)$$

if

$$\frac{F_1}{F_0^{3/2}} + 2F_0^{\left(\frac{1}{N-1}\right)} < 0,$$

i.e., if

$$\frac{F_1}{F_0^{\frac{3N-1}{2(N-1)}}} < -2. \tag{25}$$

Thus there is a guarantee of formation of central singularity earlier than apparent horizon in any $(N+2)$-dimensional space if the above inequality holds.

(ii) If the first non-vanishing derivative of density is ρ_2 (i.e. $n = 2$), then to the leading order Eq. (17) becomes

$$t_{ah}(r) = t_c(0) - \left(\frac{2}{N+1}\right) F_0^{\left(\frac{1}{N-1}\right)} r^{\left(\frac{N+1}{N-1}\right)}. \tag{26}$$

Since F_0 is always positive, we must have $t_{ah}(r) < t_c(0)$, leading the collapse to a black hole.

From Eq. (8), the time of formation of central singularity is given by

$$t_c(0) = \frac{2}{(N+1)\sqrt{F_0}}. \tag{27}$$

The values of $t_c(0)$ in different dimensional space-time are summarized in Table 1.

Table 1 The values of $t_c(0)$ in different dimensions.

N	D Dimension	$t_c(0)$ $F_0 = 1$	$t_c(0)$ $F_0 = 2$
2	4	0.667	0.4714
3	5	0.5	0.3536
4	6	0.4	0.2828
5	7	0.333	0.2357
6	8	0.28	0.2020
7	9	0.25	0.1768

It can be observed from the above table that the time of formation of central singularity decreases with the increase in the dimensions of spacetimes.

The graph of time of formation of central singularity, $t_c(0)$ versus dimension of the space-time, D is plotted in Fig. 1.

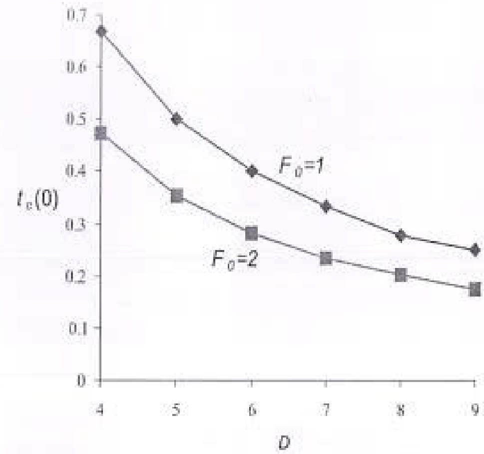

Fig. 1 The graph of $t_c(0)$ versus D.

It is interesting to note from the values given in the table that the time of formation of central singularity decreases with the increase in dimensions of the space-time. This is quite possible because gravitational force is directly propositional to the dimensions of the space-time.[9] If we increase the spatial dimension then it would increase the gravitational force of the collapsing star. Because of this additional gravitational force, the collapse has to take place earlier, which in turn decreases the time of formation of the central singularity.

In case of the gravitational collapse of spherical dust cloud with smooth analytical density function $\rho(r)$, the function $\rho(r)$ has the form

$$\rho(r) = \rho_0 + \frac{\rho_2 r^2}{2!} + \frac{\rho_4 r^4}{4!} + \cdots . \tag{28}$$

Then for $N \geq 4$ (i.e. for $D \geq 6$), Eq. (17) becomes

$$t_{ah}(r) = t_c(0) - \left(\frac{1}{N+1}\right)\frac{F_2}{F_0^{3/2}}\, r^2 - \left(\frac{2}{N+1}\right) F_0^{\left(\frac{1}{N-1}\right)}\, r^{\left(\frac{N+1}{N-1}\right)}. \tag{29}$$

In the above expression, the third term dominates the second term, and consequently the apparent horizon decreases near the center. In other words apparent horizon will form earlier than the central singularity hence the singularity will not be naked. Thus in higher-dimensional space-times where $D \geq 6$, if one chooses smooth analytical density function as an initial data then the CCH is always respected. This result is in agreement with the results obtained in Refs. [10,14].

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Final Fate of (n+2)-Dimensional Monopole Vaidya Solution.

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Abstract

We analyze here the gravitational collapse of higher-dimensional monopole Vaidya solution and we study the occurrence and nature of the naked singularities formed in the gravitational collapse of monopole Vaidya solution in higher dimensional space-times. Both naked singularities and black holes are shown to be developing as final outcome of the collapse. Earlier work is generalized to higher dimensional space-times to allow a study of the effect of number of dimensions on the possible final outcome of the collapse in terms of a black hole or a naked singularity. No restriction is adopted on the number of dimensions.

Keywords: Cosmic censorship, naked singularity, gravitational collapse.
PACS numbers: 04.20Dw, 04.20Cv, 04.70Bw

I. Introduction

The investigation on the final fate of gravitational collapse of initially regular distribution of matter is one of the most active field of research in the contemporary general relativity. The physical phenomena in astrophysics and cosmology involve gravitational collapse in a fundamental way. The final fate of a massive star, when it collapses under its own gravity at the end of its life cycle, is one of the most important questions in gravitation theory and relativistic astrophysics today. A sufficiently massive star more than five times the size of sun would undergo a continuous gravitational collapse, on exhausting its nuclear fuel, without achieving an equilibrium state such as a *neutron star or white dwarf*. The singularity theorems in general relativity then predict that the collapse gives rise to a *space-time singularity*, either hidden within an event horizon of gravity or visible to external universe [1]. The densities and space-time curvatures get arbitrarily high and diverge at these ultra strong gravity regions. Their visibility to outside observers is determined by the casual structure within the dynamically developing collapsing cloud, as governed by the Einstein field equations. When the internal dynamics of the collapse delays the horizon formation, these become visible, and may communicate physical effects to the external universe.

It is assumed that the 4-dimensional space-time of the universe we live in is obtained through a dimensional reduction from higher dimensional space-time. Inspired by work in the string theory and other field theories, there has been considerable interest in recent times to find solutions of the Einstein equation in dimensions greater than four. It is believed that underlying space-time in the large energy limit of the Planck energy may have higher dimensions than the usual four. Higher dimensional gravity theories have been considered as possible avenues to unify the basic forces of nature. 5D Kaluza-Klein [2] theory unifies gravity and electromagnetism and extensions of this have been investigated in [3]. The extra dimensions have been assumed to be small, typically of the order of the Planck length and so Kaluza-Klein Models are highly massive.

Nevertheless, the extra dimensions will not be directly observable in experiments. The success of string theories gave encouragement to search for indirect methods to detect the extra dimensions. Possible effects of the extra dimensions considered as bulk in the standard model have been suggested by Arkani-Hamid, Dimopoulos [110,111]. Higher dimensional space-time is now an active field of research in its attempts to unify gravity with all other forces of nature. It is particularly relevant in cosmology where it is shown that under certain situations, Einstein field equations dictate that as the usual 3D space expands the extra dimensions contract with time via the well- known process of dimensional reduction.

The results on gravitational collapse in higher dimensions are of interest in view of the current possibilities being explored for higher dimensional gravity. In this paper, we shall generalize our previous [4] studies to the case of monopole Vaidya solution in (n+2)-dimensional space-times. We show that the results for gravitational collapse, obtained in four-dimensional monopole space-time are also valid in (n+2)-dimensional monopole space-time. The objective is to fully investigate the situation in the background of higher dimensional space-time.

In section II and III, we give (n+2)-dimensional solution to monopole Vaidya solution. In section IV, we discuss the nature of singularity (visible or invisible), by analyzing the outgoing radial null geodesics emanating from the central singularity. We conclude the paper in V section by some concluding remarks.

II. Monopole Vaidya solution in (n+2)-dimensional space-times

Generalized Vaidya metric in (n+2)-dimensional space-time is given by [5, 6]

$$ds^2 = -Adu^2 + 2dudr + r^2 d\Omega^2 \tag{1}$$

Where u is the advanced Eddington time coordinate and r is the radial coordinate with $0 < r < \infty$ and

$$A = \left(1 - \frac{m(u,r)}{r^{n-1}}\right) \tag{2}$$

Where $m(u,r)$ gives the gravitational mass inside the sphere of radius r and

$$d\Omega^2 = d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3^2 + \dots + sin^2\theta_1 sin^2\theta_2 \dots sin^2\theta_{n-1} d\theta_n^2 \tag{3}$$

is a metric on n-sphere.

We find that corresponding energy-momentum tensor can be written as [6-8]

$$T_{\mu u} = T_{\mu u}^{(n)} + T_{\mu u}^{(m)}, \tag{4}$$

and

$$T_{\mu u}^{(n)} = \sigma l_\mu l_u , \tag{5}$$

also

$$T_{\mu u}^{(m)} = (\rho + P)(l_\mu \eta_u + l_u \eta_\mu) + P g_{\mu u} \tag{6}$$

In the comoving coordinate $(u, r, \theta_1, \theta_2, \dots, \theta_n)$, the two eigen vectors of energy-momentum tensor namely l_μ and η_u are linearly independent future pointing light-like vectors (null vectors) having components

$$l_\mu = \delta_\mu^0, \quad \eta_\mu = \frac{1}{2}\left(1 - \frac{m}{r^{n-1}}\right)\delta_\mu^0 - \delta_\mu^1 , \tag{7}$$

$$l_\mu \eta^\lambda = -1, \qquad l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0 \tag{8}$$

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Where ρ and P are the energy density and thermodynamic pressure, σ is the energy density corresponding to Vaidya null radiation.

In particular, when $\rho = P = 0$, the solution reduces to higher dimensional Vaidya solution with $m = m(u)$ [9]

Therefore for the general case we consider the EMT of equation (6).

Strong and weak energy conditions for ρ, P and σ are given by

(a) Weak and strong energy condition:

$$\sigma > 0, \quad \rho \geq 0, \quad P \geq 0. \qquad (9)$$

(b) Dominant energy conditions:

$$\sigma > 0, \quad P > 0, \quad \rho > P \qquad (10)$$

The non-vanishing component of the Einstein field equations

$$G_{\mu u} = KT_{\mu u}, \qquad (11)$$

for the metric (1) with matter field having stress-energy tensor given by (4) are

$$\rho = \frac{nm'}{k(n-1)r^n} \qquad (12)$$

$$P = \frac{-m''}{k(n-1)r^{n-1}} \qquad (13)$$

and

$$\sigma = \frac{n\dot{m}}{k(n-1)r^n} \qquad (14)$$

Where dot and dash stand for differentiation with respect to u and r respectively.

In order to satisfy the energy conditions (9) and (10), we have the following restrictions on m from the field equations (12) –(14).

(i) $m' \geq 0$, $m'' \leq 0$, (ii) $\dot{m} > 0$.

The condition (i) means that the mass function either increases with r or is constant. The second restriction (ii) implies that matter within radius r increases with time.

III. Monopole Vaidya Solution in (n+2) dimensions Space –time

Mass function given by A. Wang in case of 4D is given by

$$m(u,r) = \alpha r \qquad 0 < \alpha < 1$$

Where α is arbitrary constant.

Mass function in (n+2) dimensional Monopole Vaidya solution is given by

$$m(u,r) = \alpha r^{n-1} \qquad 0 < \alpha < 1 \qquad (15)$$

Monopoles are formed due to guaze symmetry breaking and have many properties of elementary particles. Most of their energy is concentrated in a small region near Monopole score.

Following references [7, 8], we define the mass function of Vaidya space-time in (n+2)-dimension as

$$m(u,r) = g(u) \qquad (16)$$

Where $f(v)$ and $g(v)$ are arbitrary functions which are restricted by the energy Conditions [7,8].

Since the energy momentum tensor linear in terms of mass functions, a linear superposition of particular solutions is also a solution of Einstein's field equation (1) in particular combining the mass functions (15) and (16) ,we obtain the mass function for Monopole Vaidya solution as

$$m(u,r) = \alpha r^{n-1} + g(u) \qquad (17)$$

The physical situation is for $u < 0$; the space-time is (n+2)-dimensional Minkowskian monopole field with $g(u) = 0$. The radiation is focused into central singularity at $r = 0, u = 0$ of growing mass $g(u)$. At $u = T$, say, the radiation is turned off.

For $u > T$, the exterior space-time settles into (n+2)-dimensional Reissner-Nordstorm monopole field solution.

Hence using mass function (17), (n+2)-dimensional Vaidya monopole space-time (1) can be written as

$$ds^2 = -\left[1 - \left(\frac{\alpha r^{n-1}}{r^{n-1}} + \frac{g(u)}{r^{n-1}}\right)\right] du^2 + 2dudr + r^2 d\Omega^2$$

$$ds^2 = -\left[1 - \alpha - \frac{g(u)}{r^{n-1}}\right] du^2 + 2dudr + r^2 d\Omega^2 \quad (18)$$

The above metric may also be called as (n+2)-dimensional generalized Vaidya monopole metric.

IV. Nature of the Singularity

We now investigate the nature of singularity in both asymptotically flat and cosmological solution. To investigate the nature of singularity, we follow the method given in references [9]. The central singularity is said to be naked, if the radial null geodesic equation admits at least one real and positive root [9, 10].

The outgoing radial null geodesic equation for the metric (18) is given by

$$\frac{dr}{du} = \frac{1}{2}\left[1 - \alpha - \frac{g(u)}{r^{n-1}}\right] \qquad (19)$$

In general the above equation does not yield analytic solution for $g(u)$.

However, if one chooses, $g(u) \propto u^{n-1}$, then the equation (17) becomes homogeneous and can be solved in terms of the elementary function [65].

Therefore let us choose

$$g(u) = \lambda u^{n-1} \qquad (20)$$

With the above mass function, Vaidya space-time metric (18) becomes

$$ds^2 = -\left[1 - \alpha - \frac{\lambda u^{n-1}}{r^{n-1}}\right] du^2 + 2dudr + r^2 d\Omega^2 \quad (21)$$

To investigate the structure of the collapse, we need to consider the radial null geodesics defined by $ds^2 = 0$, taking $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = \cdots \dot{\theta}_n = 0$ into account.

For metric (21), the radial null geodesics must satisfy the null condition

$$\frac{du}{dr} = \frac{2}{\left[1 - \alpha - \frac{\lambda u^{n-1}}{r^{n-1}}\right]} \qquad (22)$$

However, we recall that the coordinate v is an advanced time coordinate. It can be observed from equation (22) that this equation has a singularity at $r \to 0$, $u \to 0$. In order to classify the radial and non-radial outgoing non-space like geodesics terminating at this singularity in the past, we need to consider the limiting value of $X = \frac{u}{r}$ along a singular geodesic as the singularity is approached [10].

Thus, for the geodesic tangent to exist uniquely at this point, we must have that

$$X_0 = \lim_{\substack{u \to 0 \\ r \to 0}} \frac{u}{r} = \lim_{\substack{u \to 0 \\ r \to 0}} \frac{du}{dr} \quad (23)$$

$$X_0 = \lim_{\substack{u \to 0 \\ r \to 0}} \frac{2}{\left[1 - \alpha - \frac{\lambda u^{n-1}}{r^{n-1}}\right]} \quad (24)$$

$$X_0 = \frac{2}{1 - \alpha - \lambda X_0^{n-1}} \quad (25)$$

i.e. $X_0 - \alpha X_0 - \lambda X_0^n = 2$

$$\lambda X_0^n + \alpha X_0 - X_0 + 2 = 0$$

$$\lambda X_0^n - (1 - \alpha) X_0 + 2 = 0 \qquad 0 < \alpha < 1 \quad (26)$$

The above algebraic equation decides the nature of the singularity. If it has a real and positive root, then there exists future directed radial null geodesics originating from $r = 0$ *and* $u = 0$. In this case, the singularity will be naked. If this equation has no positive root, then the singularity will be covered and the collapse ends into a black hole.

To study the equation (26), we shall consider some different values of n, λ, *and* α.

Case 1: Let us take $n = 2$, and then monopole space-time (21) reduces to four dimensional Vaidya monopole space-time. This four dimensional Vaidya monopole solution admits strong naked singularities.

In particular if one chooses $n = 2$, $\alpha = 0.5$, then equation (26) reduces to

$$\lambda X_0^2 - 0.5 X_0 + 2 = 0 \quad (27)$$

One can easily check that for $\lambda = 0.01$, one of the roots of equation (27) is $X_0 = 4.3845$, which ensures that the singularity is naked.

Case 2: For $n = 3$, the space-time (21) reduces to five dimensional monopole Vaidya solution given by

$$ds^2 = -\left[1 - \alpha - \frac{\lambda u^2}{r^2}\right] du^2 + 2dudr + r^2[d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2] \quad (28)$$

This five dimensional monopole Vaidya solution admits strong naked singularity.

In particular if one chooses $n = 3$, $\alpha = 0.5$, then one of the positive real roots of equation (26) reduces to

$$\lambda X_0^3 - 0.5 X_0 + 2 = 0 \quad (29)$$

If we consider $\lambda = 0.001$, $\alpha = 0.5$, then one of the positive real root of equation (29) is $X_0 = 4.14213$ which shows that five dimensional monopole Vaidya solution has a naked singularity.

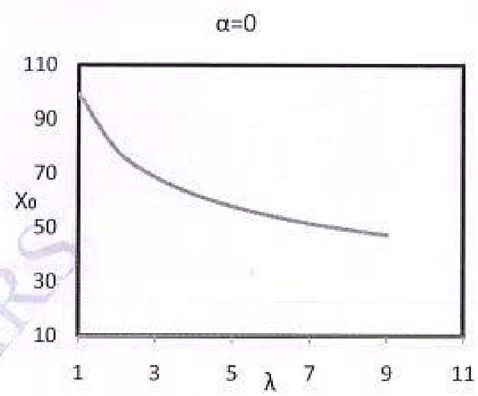
For n=4 (i.e. for 6D), the equation (26) becomes,

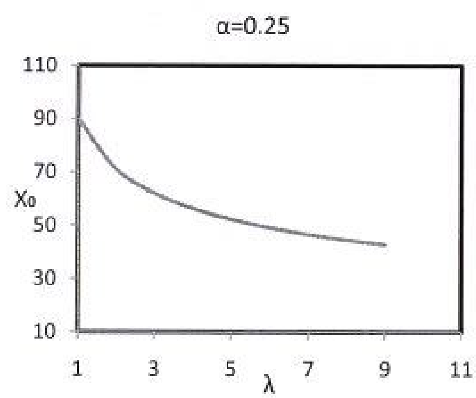
$$\lambda X_0^4 - (1 - \alpha) X_0 + 2 = 0 \quad (30)$$

then the roots of equation (30) obtained for different values of λ in six dimensional monopole Vaidya solution are shown in the following table.

Table 2.1 Values of X_0 for different values of λ in 6D.

λ	X_0			
	α=0	α=0.25	α=0.5	α=0.75
$1 * 10^{-6}$	99.3242	89.9492	77.9892	60.065
$2 * 10^{-6}$	78.6919	71.2007	61.6019	46.9839
$3 * 10^{-6}$	68.6562	62.0808	53.6284	[illegible]
$4 * 10^{-6}$	62.3148	56.3177	48.5885	36.548
$5 * 10^{-6}$	57.7979	52.2125	44.9976	33.6534
$6 * 10^{-6}$	54.3486	49.0775	42.2548	31.435
$7 * 10^{-6}$	51.5914	46.5713	40.0617	29.6552
$8 * 10^{-6}$	49.3147	44.5019	38.2505	28.1802
$9 * 10^{-6}$	47.389	42.7513	36.7179	26.9277





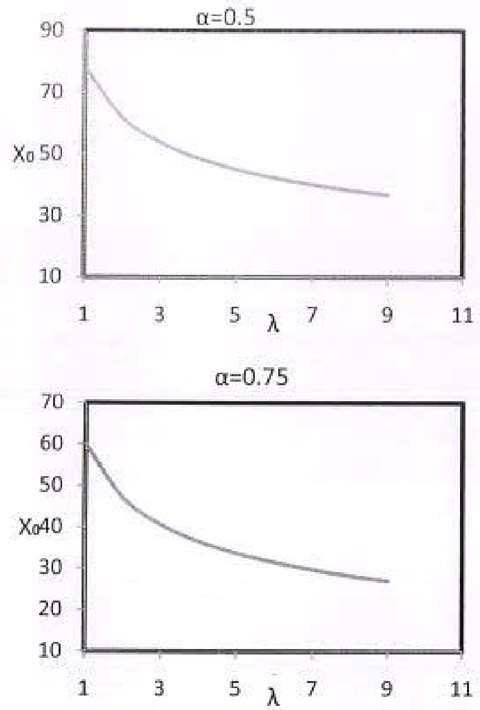


Figure 2.1: Graph of the values of X_0 against the values of λ.

From the graph we may observe that initially the value of X_0 is at peak and the values of X_0 decreases when increase the value of λ. It is observe that the peak shifted towards lower values of X_0 for the different values of α. As we increase the values of α the values of X_0 decreases.

For $n = 5$ $(i.e.\ for\ 7D)$, the equation (2.26) becomes

$$\lambda X_0^5 - (1-\alpha)X_0 + 2 = 0 \qquad (31)$$

then the roots of equation (31) obtained for different values of λ in seven dimensional monopole Vaidya solution are shown in the following table.

Table 2.2 Values of X_0 for different values of λ in 7D.

λ	X_0			
	α=0	α=0.25	α=0.5	α=0.75
$1 * 10^{-7}$	55.722	51.6427	46.2292	37.4446
$2 * 10^{-7}$	46.773	43.312	38.6935	31.0358
$3 * 10^{-7}$	42.213	39.0669	34.8519	27.7513
$4 * 10^{-7}$	39.247	36.3051	32.3518	25.6032
$5 * 10^{-7}$	37.088	34.2953	30.5319	24.0321
$6 * 10^{-7}$	35.412	32.7342	29.1179	22.8057
$7 * 10^{-7}$	34.052	31.4686	27.9713	21.8064
$8 * 10^{-7}$	32.917	30.4109	27.0128	20.9671
$9 * 10^{-7}$	31.946	29.5067	26.1933	20.2459

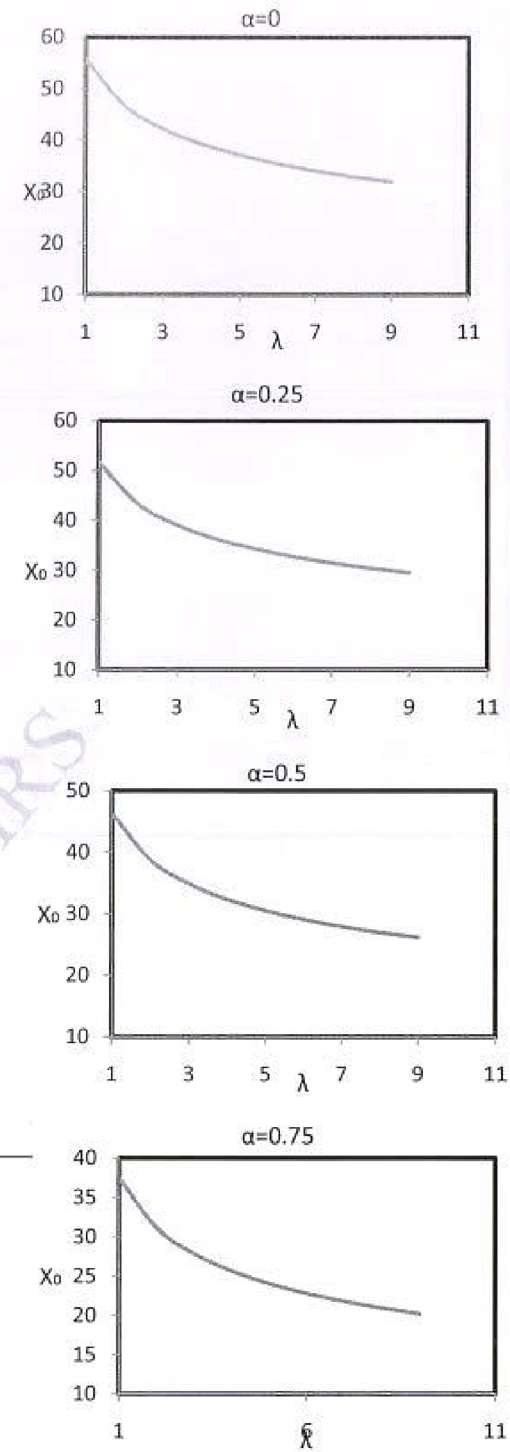


Figure 2.2: Graph of the values of X_0 against the values of λ.

25

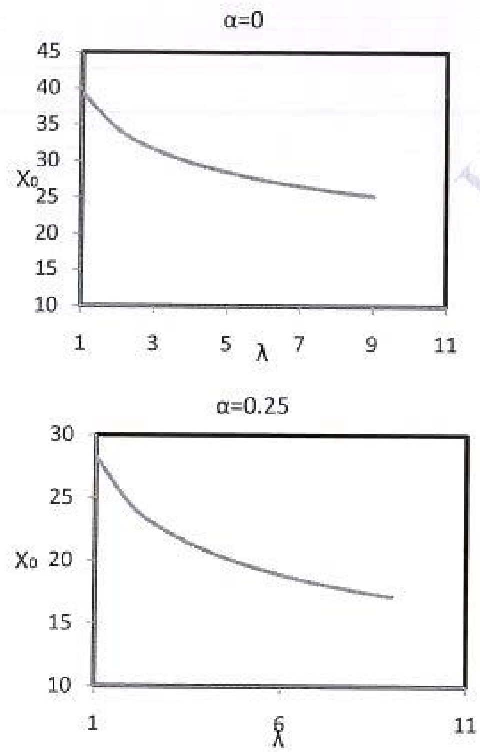
If we observe the above graph we see that in seven dimensional space-time , the values of X_0 decrease as we increase the value of λ . Also for increasing α the values of X_0 decreases very rapidly.
Similarly for $n = 6$ $(i.e. for\ 8D)$, the equation (2.26) becomes

$$\lambda X_0^6 - (1-\alpha)X_0 + 2 = 0 \qquad (32)$$

then the roots of equation (32) obtained for different values of λ in seven dimensional monopole Vaidya solution are shown in the following table.

Table 2.3 Values of X_0 for different values of λ in 8D .

λ	X_0			
	α=0	α=0.25	α=0.5	α=0.75
1	39.398	28.2253	33.795	28.2254
2	34.242	24.2437	29.298	24.2437
3	31.541	22.1423	26.9407	22.1423
4	29.753	20.7421	25.3796	20.7421
5	28.436	19.7038	24.2286	19.7038
6	27.402	18.8841	23.3251	18.8841
7	26.557	18.2101	22.5862	18.2101
8 *	25.845	17.6394	21.9642	17.6394
9	25.233	17.1455	21.4289	17.1455



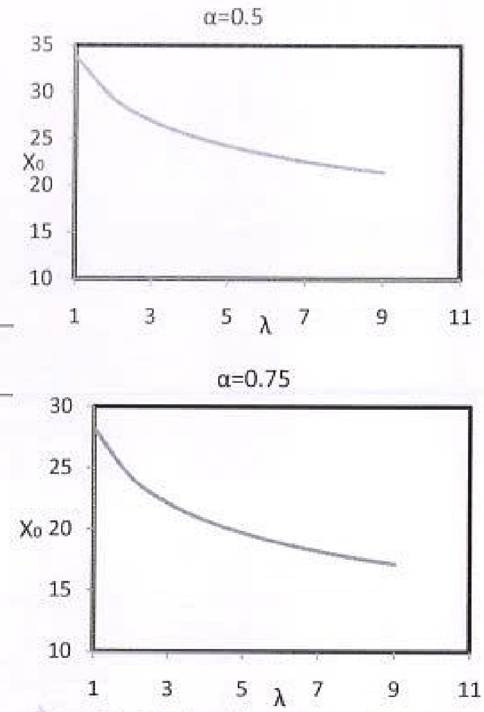


Figure 2.3: Graph of the values of X_0 against the values of λ .

From the graph we may observe that in eight dimensional space-time X_0 decreases as we increase the value of λ. Also observing that for the higher dimensions the values of X_0 decreases as well as the values of λ .

Case 3:- For constant α ,

if $\alpha = 0$ the equation (26) becomes

$$\lambda X_0^n - X_0 + 2 = 0 \qquad (33)$$

then the roots of equation (38) obtained for different values of λ in seven dimensional monopole Vaidya solution are shown in the following table.

Table 2.9 Values of X_0 for different values of λ for $\alpha = 0$.

λ	X_0			
	n=4	n=5	n=6	n=7
$1 * 10^{-6}$	99.3242	31.1017	15.4145	9.619
$2 * 10^{-6}$	78.6919	26.066	13.3569	8.5205
$3 * 10^{-6}$	68.6562	23.4997	12.2781	7.9333
$4 * 10^{-6}$	62.3148	21.8299	11.5636	7.5395
$5 * 10^{-6}$	57.7979	20.6147	11.0367	7.2465
$6 * 10^{-6}$	54.3486	19.6707	10.6231	7.0148
$7 * 10^{-6}$	51.5914	18.9054	10.2848	6.8241
$8 * 10^{-6}$	49.3147	18.2657	10	6.6627
$9 * 10^{-6}$	47.3889	17.7189	9.7549	6.5231

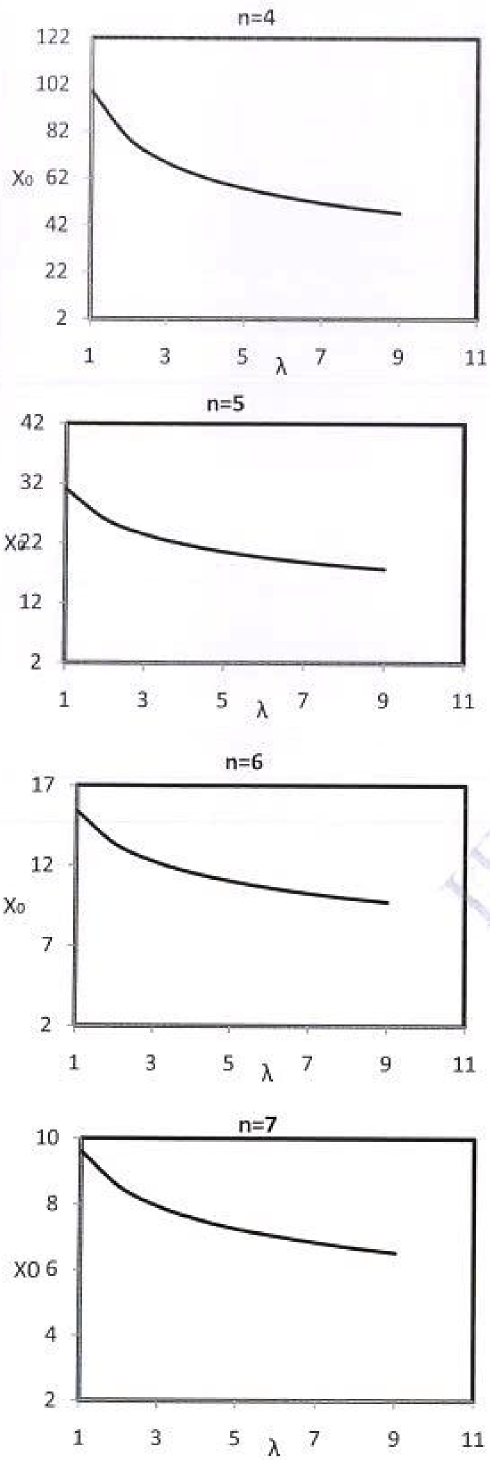


Figure 2.9: Graph of the values of X_0 against the values of λ for fixed value of α .

From the graph we may observe that for fixed α and λ the values of X_0 decreases very rapidly in higher and higher dimensions. Initially the value of X_0 is at peak then decreases when increase the value of λ .

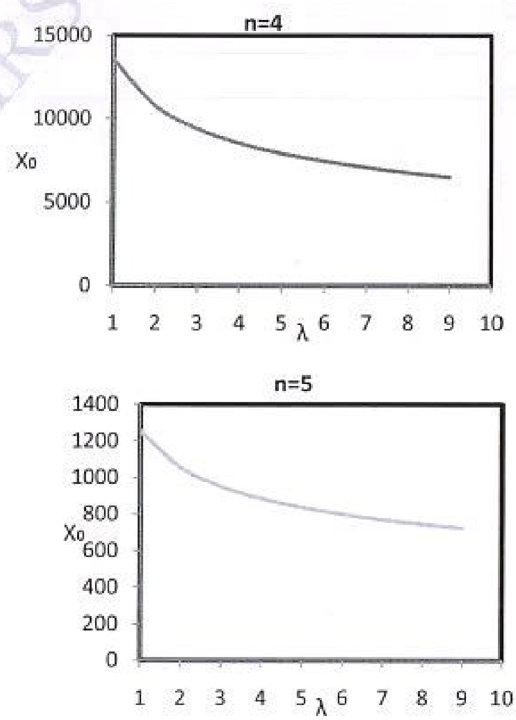
For $\alpha = 0.75$, the equation (26) becomes

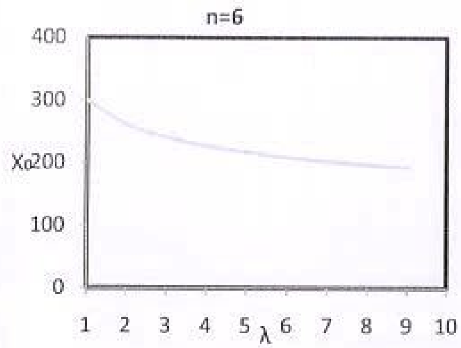
$$\lambda X_0^n - 0.25X_0 + 2 = 0$$
(34)

then the roots of equation (41) obtained for different values of λ for $\alpha = 0.75$ monopole Vaidya solution are shown in the following table.

Table 2.12 Values of X_0 for different values of λ for $\alpha = 0.75$.

λ	X_0			
	n=4	n=5	n=6	n=7
1	13569.42	1255.42	300.082	115.109
2	10769.50	1055.36	261.022	102.391
3	9407.692	953.432	240.561	95.6044
4	8547.211	887.128	227.017	91.0592
5	7934.336	838.884	217.036	87.6805
6	7466.339	801.415	209.204	85.0115
7	7092.248	771.042	202.802	82.8172
8	6783.375	745.660	197.413	80.9613
9	6522.110	723.965	192.778	79.3582





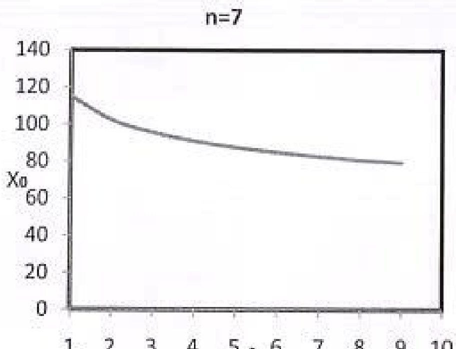


Figure 2.12: Graph of the values of X_0 against the values of λ for fixed value of α.

From the graph we may observe that when we increase the value of α the value of X_0 increases very rapidly and X_0 decreases for higher and higher dimensions of same α.

V. Concluding Remarks

Gravitational collapse is one of the most fruitful subjects in gravitational physics. It is well known that singularity formation is inevitable in complete gravitational collapse.It was conjectured that such a singularity should be hidden by horizons if it is formed from generic initial data with physically reasonable matter fields [1].

Many possible counter examples to this conjecture have been proposed over the past four decades[11,12,13], although none of them have proved to be sufficiently generic [14,15]. In these examples, there appears a singularity that is not hidden by horizons. This singularity is called a *naked singularity*.

In the present work shows that naked singularities do occur as the end stage of gravitational collapse in (n+2) dimensional Monopole Vaidya solution and shown that the result in 4-dimensional space-time are also valid in (n+2)-dimensional Monopole Vaidya solution. Imposing some conditions on mass function ,it has been shown that the singularities arising (n+2)-dimensional Monopole Vaidya solution are naked in any arbitrary dimensions.

It is also note that the roots of equation is decreases for increasing the value of α for higher and higher dimensions where as for fix α the roots of equation is increases for higher dimensions. In this sense, both black holes and naked singularities do seem to arise generically as the end product for (n+2) dimensional Monopole Vaidya solution.

Thus which is seems suggest that the dimension of the space-time does not play any fundamental role in the formation of the naked singularities. Occurrence of naked singularities in higher dimensional Vaidya monopole space-time suggests that the higher dimensional Vaidya monopole solution violates the cosmic censorship hypothesis. These results might be important in the light of the recent proposal that there may exist extra dimensions in the universe.

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International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 11, November - 2013

Monopole-Radiating Dyon Solution in Anti-De Sitter Space Time

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Abstract

We analyze here the structure of space-time singularities formed during the radial in-fall of a coherent stream of charged "photons"-a piece of the monopole radiating dyon metric. we study the nature of singularities which develop in the space time on the anti-de-sitter background. It is shown that the singularities formed in gravitational collapse of monopole-radiating dyon solution in anti-de-sitter background are not hidden inside the event horizon. It is also shown that final outcome of collapse depends sensitively on the electric and magnetic charge parameters. Further it is found that naked singularities are strong in Tripler's sense.

I Introduction.

It would be interesting to investigate whether the singularity forming at the end of gravitational collapse is observable. There is an important conjecture related to the singularities known as cosmic censorship hypothesis (CCH) given by Penrose [1]. This states that the collapse of a physically reasonable initial data yields a space-time singularity which is always hidden behind the event horizon. It has two versions, i.e., weak and strong. According to the weak version, singularity formed by gravitational collapse is not visible to a far away observer. The strong cosmic censorship hypothesis states that the singularity cannot be observed even by an observer who is very close to it. Wald [2] discussed some examples to justify the validity of weak form of CCH.

When a massive star is on the verge of completing its nuclear cycle, then the thermonuclear reactions in the interior of the star cannot counter balance the immense gravitational pull of the star. Under most general conditions general relativity predicts that such a collapse must end in a singularity, which may or may not be clothed by an event horizon. A singularity may be physically described as a region in the space-time with extreme curvature, vanishing volume and unbounded gravitational forces. However, general relativity remains silent on the nature (BH or NS) or physical properties of such a singularity. This is basically due to the fact that mathematical structure breaks down preventing analysis at and beyond the singularity. This has triggered extensive research on Gravitational collapse during the past few decades. After all one would always like to know whether, and under what conditions gravitational collapse leads to the formation of a black hole (BH). A few decades back R. Penrose (1969) proposed the cosmic censorship hypothesis (CCH), which states that the singularities formed in gravitational collapse of physically reasonable matter cannot be seen by any distant observer in the universe. It implies that the singularities formed in asymptotically flat space-times are always bounded by event horizons and hence are destined to be black holes. With the announcement of this proposal, study of gravitational collapse has gained special importance, because one would always like to know that whether there exists any physical collapse solutions that lead to naked singularities (NS), which will serve as counterexamples of CCH [3]. Important cases of naked singularities analyzed so for include dust collapse [4-9], radiation collapse [10-17], collapse of perfect fluid [18,19] and strange quark matter [20-21].

A.Chamorro and K.S.Virbhadra have obtained an exact solution of the Einstein- Maxwell equations which are a magnetic charge generalization to the Bonnor-Vaidya solution and describe the gravitational and electromagnetic fields of a non-radiating massive radiating dyon [22]. The paper is based on the composite charges i.e. an electric charge and a magnetic charge bound together by their gravitational interaction. Hence it would be interesting to study the nature of the singularities formed in the gravitational collapse of such composite space-time [23]. In this paper, we study the Monopole-Radiating Dyon solution in anti-de-sitter space-time. We also show that gravitational constant does affect the nature of singularity.

We conclude the paper in V section by some concluding remarks.

II Monopole-Radiating Dyon Solution.

The metric, which describes the gravitational field of non-rotating massive radiating dyon as found by Chamorro and Virbhadra [21] is

$$ds^2 = -\left(1-\frac{2m(u,r)}{r}\right)du^2 + 2dudr + r^2(d\theta^2 + sin^2\theta d\phi^2), \quad (1)$$

Where

$$m(u,r) = f(u) + g(u,r) - \frac{\left(q_e^2(u)+q_m^2(u)\right)}{2r} - \frac{\Lambda r^3}{3} \quad (2)$$

Here $f(u)$ is the standard Vaidya mass, $g(u,r)$ is monopole function, $q_e(u)$ and $q_m(u)$ are electric and magnetic charge parameters respectively. These parameters depend on the Eddington advanced time coordinate u.

The model considered in this paper is obtained from an energy-momentum tensor of the form

$$G_i^k = R_i^k - \frac{1}{2}Rg_j^k = 8\pi(E_i^k + N_i^k) \quad (3)$$

Where

$$\{x^\mu\} = \{u,\ r,\ \theta,\ \phi\}, \qquad (\mu = 0,\ 1,\ 2,\ 3).$$

E_i^k is related to the electromagnetic tensor F_{ki} in the familiar way

$$E_i^k = \frac{1}{4\pi}\left[-F_{im}F^{km} + \frac{1}{4}g_i^k F_{mn}F^{mn}\right] \quad (4)$$

$$N_i^k = V_i V^k \quad (5)$$

is the energy-momentum tensor of the null fluid. V^k is the null fluid current vector satisfying $g_{ik}V^iV^k = 0$. Electric current vector $J_{(e)}^i$ and magnetic current vector $J_{(m)}^i$ are given by

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left(\sqrt{-g}F^{ik}\right) = 4\pi J_{(e)}^i \quad (6)$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left(\sqrt{-g}\,{}^*F^{ik}\right) = 4\pi J_{(m)}^i \quad (7)$$

Where ${}^*F^{ik}$ is the dual of the electromagnetic field tensor F^{ik} and is given by

$${}^*F^{ik} = \frac{1}{2\sqrt{-g}}\in^{iklm} F_{lm} \quad (8)$$

Where $\in^{iklm}$ is the Levi-Civita tensor density.

The non-vanishing components of the Einstein tensor for the above metric are given by

$$G_0^0 = G_1^1 = -G_2^2 = -G_3^3 = \frac{\left(q_e^2(u)+q_m^2(u)\right)}{r^4}, \quad (9)$$

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$$G_0^1 = k^2 , \quad (10)$$

Where

$$k^2 = \frac{2(q_e \dot{q}_e + q_m \dot{q}_m - \dot{f} r)}{r^3} \quad (11)$$

Here the over dot denotes the derivative with respect to the retarded coordinate u.

Energy-momentum tensor of the electromagnetic field and null fluids are given by

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{\left(q_e^2(u) + q_m^2(u)\right)}{8\pi r^4} , \quad (12)$$

$$N_0^1 = \frac{k^2}{8\pi} \quad (13)$$

III Nature of the Singularities in Monopole-Radiating Dyon Solution.

The physical solution is for < 0 , the space-time becomes flat with $f(u) = 0$, $q_e(u) = 0$, $q_m(u) = 0$. At $u = T$, Say, the radiation is turned off. For$u > T$, $\dot{f}(u) = \dot{q_e}(u)\, \dot{q_m}(u) = 0$, i.e. $f(u), q_e^2(u)$ and $q_m^2(u)$ are positive definite . Thus the metric for $u = 0$ to $u = T$ is radiating dyon solution and for $u > T$ it becomes a static dyon solution [24].

To investigate the structure of the collapse, we need to consider the radial null geodesics define by $ds^2 = 0$. $(k^\theta = k^\phi = 0)$. The equation for outgoing radial null geodesic for metric (1) is given by

$$\left(1 - \frac{2m(u,r)}{r}\right) du^2 - 2dudr = 0$$

i.e.

$$\frac{dr}{du} = \frac{1}{2}\left(1 - \frac{2m(u,r)}{r}\right).$$

Using the mass function (2) , above equation becomes

$$\frac{dr}{du} = \frac{1}{2}\left(1 - \frac{2f(u)}{r} - \frac{2g(u,r)}{r} + \frac{q_e^2(u) + q_m^2(u)}{r^2} + \frac{\Lambda r^2}{3}\right) \quad (14)$$

In general, Eq. (14) does not yield an analytic solution. However, if $f(u) \propto u$, $q_e^2(u) \propto u^2$, $q_m^2(u) \propto u^2$,Eq. (14) becomes homogeneous and can be solved in terms of elementary functions[25].

In particular, we take

$$f(u) = \lambda u, \qquad g(u,r) = ar \quad (15)$$

and

$$q_e^2(u) = \begin{cases} 0 , & u \le 0 \\ \frac{\mu^2 u^2}{2} , & 0 < u \le T \\ \mu^2 T^2 (const) , & u > T \end{cases} \quad (16)$$

$$q_m^2(u) = \begin{cases} 0 , & u \le 0 \\ \frac{\delta^2 u^2}{2} , & 0 < u \le T \\ \delta^2 T^2 (const) , & u > T \end{cases} \quad (17)$$

Where λ, a, μ^2 and δ^2 are some positive constants. Inserting the expressions for $f(u)$, $g(u,r)$, $q_e^2(u)$ and $q_m^2(u)$ into Eq.(2) we obtain the mass function for monopole radiating dyon solution as

$$m(u,r) = \lambda u + ar - \frac{\mu^2 u^2 + \delta^2 u^2}{4r} - \frac{\Lambda r^3}{6} \quad (18)$$

It follows that with the choice of above mass function, the metric (1) becomes self-similar [1] (a spherically symmetric space-time is a self similar if $g_u(ct, cr) = g_u(t,r)$ and $g_{rr}(ct, cr) = g_{rr}(t,r)$ for every > 0) admitting a homothetic killing vector ξ^a given by

$$\xi^a = u\frac{\partial}{\partial u} + r\frac{\partial}{\partial r} \quad (19)$$

and satisfies

$$L_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = 2g_{ab} , \quad (20)$$

Where L denote the Lie derivative.

Defining $k^a = dx^a/dk$ as a tangent to radial null geodesics, where k is an affine parameter, it follows that $\xi^a k_a$ is constant along radial null geodesics. Thus

$$\xi^a k_a = uk_u + rk_r = C, \quad (21)$$

Where C is a constant. Radial null geodesic equations of metric (1), on using the null condition $k^a k_a = 0$, takes the simple form

$$\frac{dk^u}{dk} - \left(\frac{m'}{r} - \frac{m^2}{r^2}\right)(k^u)^2 = 0, \quad (22)$$

$$\frac{dk^r}{dk} + \left(\frac{\dot{m}}{r} - \frac{m'}{r} + \frac{m^2}{r^2} + \frac{2mm'}{r^2} - \frac{2m^2}{r^3}\right)(k^u)^2 + 2\left(\frac{m'}{r} - \frac{m^2}{r^2}\right)k^u k^r = 0 \quad (23)$$

Let

$$k^u = \frac{du}{dk} = \frac{P(u,r)}{r}, \quad (24)$$

Then from the null condition $k^a k_a = 0$ we obtain

$$k^r = \left(1 - \frac{2m}{r}\right)\frac{P}{2r}, \quad (25)$$

Also

$$k_r = k_1 = g_{1j} k^j = g_{10} k^0$$

$$k_r = \frac{P}{r} \quad (26)$$

And

$$k_u = k_0 = g_{0j}k^j = g_{00}k^0 + k^1$$

Therefore

$$k_u = -\frac{P}{2r}\left(1 - \frac{2\lambda u}{r} - 2a + \frac{\mu^2u^2+\delta^2u^2}{2r^2} + \frac{\Lambda r^2}{3}\right) \quad (27)$$

Eq. (22), (27) and (28) yields

$$P = \frac{12C}{12+(12-2\Lambda r^2-6)X+12\lambda X^2-3(\mu^2+\delta^2)X^3} \quad (28)$$

Where X is a self-similarity variable defined by $X = u/r$. The singularity occurring at $r = 0$ is naked if the outgoing radial null geodesic equation has atleast one real positive root [26]. In the case of pure Vaidya space-time it has been shown that for a mass function $m(u) = \lambda u/2$, the central singularity is naked for $\lambda \leq 1/8$, and the collapse ends into black hole if $\lambda > 1/8$ [27].

Hence it would be interesting to investigate whether the gravitational collapse of Vaidya space-time could yield a naked singularity under the influence of the gravitational constant and composite field produced by electric and magnetic charges.

With the help of Eq. (18), the equations of the outgoing radial null geodesics for the metric (1) are given by

$$-\left(1-\frac{2m}{r}\right)du^2 + 2dudr = 0$$

Therefore

$$\frac{dr}{du} = \frac{1}{2}\left(1-\frac{2m}{r}\right) \quad (29)$$

i.e.

$$\frac{dr}{du} = \frac{1}{2}\left(1 - \frac{2\lambda u}{r} - 2a + \frac{\mu^2u^2+\delta^2u^2}{2r^2} + \frac{\Lambda r^2}{3}\right) \quad (30)$$

Let $$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{u}{r} = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} \quad (31)$$

Hence Eq. (31) can be written as

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} = \frac{2}{1-2\lambda X-2a+\frac{\mu^2}{2}X^2+\frac{\delta^2}{2}X^2+\frac{\Lambda r^2}{3}}$$

i.e. $$X_0 = \frac{12}{6-12\lambda X-12a+3(\mu^2+\delta^2)X^2+2\Lambda r^2}$$

$$3(\mu^2+\delta^2)X^3 - 12\lambda X^2 + (6-12a+2\Lambda r^2)X - 12 = 0 \quad (32)$$

The above equation governs the nature of the singularity. If this equation has at least one real and positive root, then the singularity will be naked. If the equation has no positive root, then the collapse ends into a black hole.

In particular, for $\lambda = 0.1$, $\mu = 0.1$, $\delta = 0.4$, $a = 0$, $\Lambda = 0.1$, $r = 0.1$, one of the roots of Eq. (32) is $X_0 = 2.0954$, indicating that the gravitational collapse in this case ends into a naked singularity.

We calculated the value of X_0 for different values of λ, μ, δ, a, Λ *and* r

For fixed $a = 0$, $\delta = 0.4$, $\Lambda = 0.1$, $r = 0.1$ then Eq.(33) becomes

$$3(\mu^2+\delta^2)X^3 - 12\lambda X^2 + (6+2\Lambda r^2)X - 12 = 0 \quad (33)$$

Table 1 Values of X_0 for different values of λ, *and* μ

λ	X_0					
	μ=0.	0.2	0.3	0.4	0.5	0.6
0.1	2.09	1.999	1.877	1.752	1.63	1.53
0.2	3.30	2.919	2.533	2.22	1.97	1.78
0.3	5.72	4.782	3.802	3.044	2.52	2.16
0.4	8.33	6.977	5.471	4.220	3.31	2.70
0.5	10.8	9.145	7.196	5.528	4.27	3.38
0.6	13.3	11.26	8.903	6.853	5.27	4.13
0.7	15.8	13.36	10.58	8.172	6.30	4.92
0.8	18.2	15.43	12.25	9.479	7.32	5.71
0.9	20.6	17.49	13.90	10.77	8.33	6.51

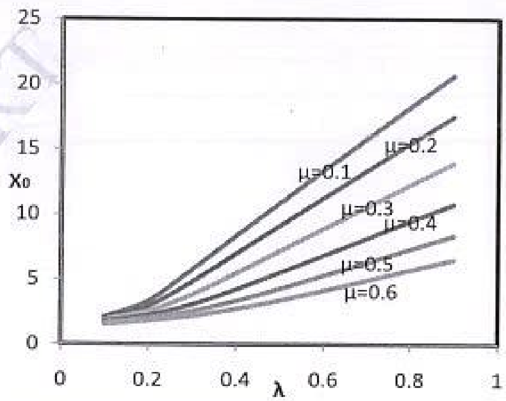

Figure 1: Graph of the values of X_0 against the value of λ.

From the graph we may observe that the value of X_0 have positive real roots. Also it is seen that the value of X_0 decreases as increase the value of μ. Also we notice that the lower value of X_0 shifted towards the peak.

For fixed value of μ and different value of λ, a we have different positive real root of X_0.

For fixed $\mu = 0.1$, $\delta = 0.4$, $\Lambda = 0.1$, $r = 0.1$ then the Eq. (32) becomes,

$$3((0.1)^2+\delta^2)X^3 - 12\lambda X^2 + (6-12a+2\Lambda r^2)X - 12 = 0 \quad (34)$$

Table 2: Values of X_0 for different values of λ *and* a

λ	X_0				
λ	a=0	0.25	0.5	0.75	0.85
0.1	2.0954	3.0025	3.8994	4.6819	4.9652
0.2	3.3002	4.549	5.4868	6.2491	6.5214
0.3	5.7206	6.7043	7.479	8.1367	8.3768
0.4	8.3388	9.0486	9.6633	10.2129	10.4186
0.5	10.8819	11.4298	11.9297	12.3923	12.5686
0.6	13.369	13.8148	14.2335	14.6294	14.7822
0.7	15.8207	16.1969	16.5562	16.9008	17.0349
0.8	18.2493	18.5748	18.8893	19.1937	19.3128
0.9	20.662	20.9491	21.2285	21.5008	21.6078

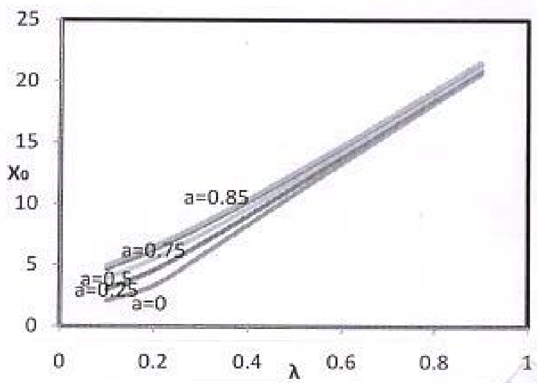

Figure 2: Graph of the values of X_0 against the value of λ for μ=0.1.

From the graph we may observe that the value of X_0 increases as increasing the value of λ.

The lower value shifted towards peak of the graph and the value of X_0 for a=0.75 and a=0.85 have the nearly same value.

For fixed value of λ and different value of μ, a we have different value of X0.

For fixed $\lambda = 0.1$, $\delta = 0.4$, $\Lambda = 0.1$, $r = 0.1$ then Eq. (32) becomes,

$$3(\mu^2 + \delta^2)X^3 - 12(0.1)X^2 + (6 - 12a + 2\Lambda r^2)X - 12 = 0 \quad (35)$$

Table 3: Values of X_0 for different values of μ, a and $= 0.1$.

μ	X_0				
μ	a=0	0.25	0.5	0.75	0.85
0.1	2.0954	3.0025	3.8994	4.6819	4.9652
0.2	1.9995	2.7837	3.5691	4.2684	4.5239
0.3	1.8772	2.5247	3.1807	3.7786	3.9996
0.4	1.7525	2.2812	2.8205	3.3224	3.5102
0.5	1.6366	2.0713	2.5161	2.9368	3.0959
0.6	1.5329	1.8953	2.2663	2.6216	2.7571
0.7	1.4416	1.7482	2.062	2.3653	2.4819
0.8	1.3612	1.6246	1.8938	2.1556	2.2568
0.9	1.2904	1.5195	1.7534	1.982	2.0707

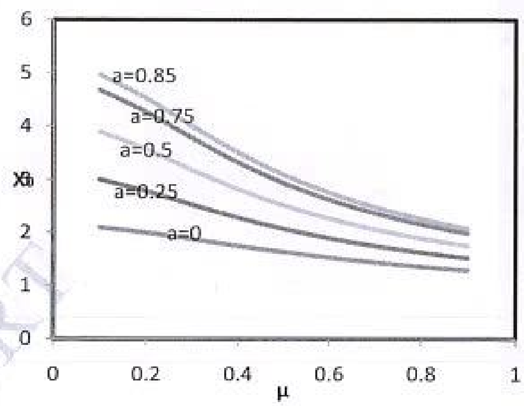

Figure 3: Graph of the values of X_0 against the value of μ for λ =0.1.

From the graph it is clear that the value of X_0 decreases for increase the value of μ. It is also note that the value of X_0 for μ=0.1 have much more difference than the value of μ=0.9 and high value shifted towards lower value.

IV Strength of the naked singularity.

It has seen the nakedness of the singularity in the previous section; in this section we study the strength of singularity. The Clark and Krolak Criterion the strength of singularities has been analyzed and shown that these naked singularities are gravitationally strong. If the naked singularity is not strong then it cannot be considered as a physically reliable singularity and hence such naked singularities may not be considered as counter examples to CCH. A naked singularity is said to be strong if at least along one radial null geodesic with affine parameter k, with k = 0 at the singularity [27], one should have

$$\Psi = \lim_{k\to 0} k^2 R_{ab} k^a k^b > 0 \quad (36)$$

Where k^a is tangent to the null geodesics and R_{ab} is the Ricci tensor

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using Eq. (24) and (25) we write

$$\Psi = \lim_{k\to 0} k^2 R_{ab} k^a k^b = \lim_{k\to 0} k^2 \frac{2m}{r^2} (k^u)^2 \qquad (37)$$

$$= [4\lambda - X(\mu^2 + \delta^2) \lim_{k\to 0} \left(\frac{kP}{r^2}\right)^2 \qquad (38)$$

as singularity is approached, $k \to 0, r \to 0$ *and* $X \to X_0$ and using L-Hospital's rule, we find that

$$\Psi = \frac{4\lambda - (\mu^2+\delta^2)X_0}{1 - 2a + \frac{\Lambda r^2}{3} - 2\lambda X_0 + \frac{1}{2}(\mu^2+\delta^2){X_0}^2} \qquad (39)$$

Thus the singularity is strong if $4\lambda - (\mu^2 + \delta^2)X_0 > 0$
For our particular case (i.e. λ=0.1, μ=0.1, δ=0.4, a=0, Λ=0.1, r=0.1, $X_0 = 2.0954$)
We have $4\lambda - (\mu^2 + \delta^2)X_0 = 0.043782$. Thus the naked singularity arising in the monopole-radiating dyon solution in the anti-de-sitter space-time is a strong curvature singularity.

V Concluding Remarks.

Cosmic censorship conjecture has become a challenging and most significant open problem in a general relativity. Many possible counter examples to this conjecture have been proposed over the past four decades, although none of them have proved to be sufficiently generic. In this work, there appears a singularity that is not hidden by horizon this singularity is called a naked singularity.

In the present work we have studied monopole radiating dyon solution in anti-de-sitter space time. Here we examine the structure of space time singularities formed during the radial in fall of coherent stream of charged "photons" – a piece of the monopole radiating dyon metric.

It has been shown that the singularities formed in gravitational collapse of monopole radiating dyon solution in anti-de-sitter background are not hidden inside the event horizon. Thus one can argue that composite charged field (electric and magnetic charges) and gravitational constant does not affect to gravity and cannot prevent a naked singularity from forming completely, so that CCH actually violets.

Also, using the clark and krolak criteria [28] the strength of singularities has been analyzed and shown that the naked singularities in the composite solution in anti-de-sitter background are gravitationally strong.

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