Minor Research Project

Title

" Mathematical Modelling for Thermoelastic Behaviour of Different Solid Body Structures Subjected to Heating"

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APPENDIX-1

Report of the work done

1.Objectives of Project:

Experimentation involving Infrared Thermography , for thermoelastic studies of solid bodies was the direct method to make the study of thermoelasticity behaviour, which at times was inconvenient , time consuming ,cumbersome and could yield errors in result.

The project was done with an objective of theoretical investigation of thermoelastic behaviour of solid bodies of different geometry. It involves determination of the heat conduction, thermal displacement, thermal strain and thermal stresses in solid objects of different geometry, by application of various mathematical tools , partial differential equations, and integral transforms to the boundary value problems governing each geometrical structures and different heating conditions.

2. Whether the Objectives were achieved :---------- Yes the objectives of the project were achieved.

Mathematical modelling has been done for theoretical investigations of thermoelastic behaviour of some solid bodies of different geometry which include Rectangular plate, Annular discs, Hollow cylinder, Circular plate, sphere etc. Attempt has also been made to synthesise a polymer material Polymethyl Methacrylate and study it's thermal behaviour with the help of thermal mechanical analyser.

3.Achievements from the project:

i) A necessary theory for mathematical modelling to study thermoelastic behaviour of geometrical bodies like Rectangular plate, Annular discs, Hollow cylinder, Circular plate, sphere etc. has been developed.
ii) This theoretical approach enables us to study the stress and strains in solid bodies without performing the experiment, and , even without actually incorporating the solid structure in heavy machinery engineering, aerospace engineering ,spacecraft and etc.)
iii)The Thermal stress bearing ability of a solid body structure can be analysed with the help of above theory.

iv) Polymer material Polymethyl Methacrylate was synthesised and it's thermal behaviour studied with the help of thermal mechanical analyser. This thermoelastic material PMMA was synthesized using wet chemical technique whose TMA study reveals the usability of the material upto 60 deg.

4. Summary of the Findings:

Mathematical modelling for thermoelastic studies has gained popularity as a means of thermal stress analysis of materials and also to test the thermal stability of the materials. The advantage of mathematical modelling is that even before a structure of particular geometry is designed for industrial engineering purpose, we can study its thermal stability and stress bearing capability.

In the chapter 1 , the temperature , displacement and thermal stresses of thin annular disc have been obtained. The finite Marchi-Zgrablich transform technique is used to obtain the numerical results. The results are obtained in terms of Bessel's function in the form of infinite series. The expressions (7.1) and (12.1) are represented graphically. Any particular case of special interest can be derived by assigning suitable values to the parameter and functions in the expression.

In chapter 2, the temperature, displacements and thermal stresses on the outer curved surface of a thin annular disc have been obtained, when the interior heat flux and the other three boundary conditions are known, with the aid of finite Marchi-Fasulo transform and Laplace transform techniques. The results are obtained in terms of Bessel's function in the form of infinite series. The series solutions converges provided if we take sufficient number of terms in the series. Here the results are expressed in terms of x & y. Since the thickness of annular disk is very small, the series solution given here will be definitely convergent.

The equation (31) is represented graphically and it is observed that as r increases the value of $g(z,t)/\alpha$ increases. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

In chapter 3 , the temperature, Displacement and stress functions at any point of the disc have been obtained, when the interior temperature and the other three boundary conditions are known, with the aids of finite Fourier sine transform, Laplace transform and Marchi-Zgrablich transform techniques. Equation (9.1) is represented graphically. The expressions are obtained in the form of infinite series. It is observed that as the value of z increases, the temperature gradually increases. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the equations.

The results presented here will be more useful in Engineering problems particularly in the determination of the state of strain in the disc which constitutes the foundations of container for hot gases or liquid and in foundations for furnaces.

In chapter 4 , we discussed completely the direct steady-state thermoelastic problem of a thin rectangular plate, where homogeneous boundary condition of the third kind is maintained on the edges y = -b, b of the rectangular plate and on the edge x = 0, the third kind boundary condition is maintained at h(y), which is a known function of y. The expression (6.1) is represented graphically. it is found that as the value of x increases, the temperature goes on decreasing. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions. The finite Marchi-Fasulo integral transform technique has been used to obtain the numerical results. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

In chapter 5, two problems are discussed. In the first problem , we discussed completely the direct unsteady-state thermoelastic problem of a thin rectangular plate on the edge x = *a* , where homogeneous boundary condition of the third kind is maintained on the edges y = -b, b and on the edge x = 0 of the rectangular plate. In the second problem , we discussed completely the direct unsteady-state thermoelastic problem of a thin rectangular plate on the edge x = *a* , where homogeneous boundary condition of the third kind is maintained on the edges y = -b, b and on the edge x = 0, the third kind boundary condition is maintained at g(y,t), which is a known function of y and t.

The finite Marchi- Fasulo integral transform and Laplace transform techniques are used to obtain the numerical results. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

In chapter 6, the temperature distribution, displacement and thermal stresses at any point of a three dimensional rectangular plate have been obtained, when the boundary conditions are known with the aid of finite Fourier cosine transform and Marchi-Fasulo transform techniques. The results are obtained in the form of infinite series. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions. The temperature distribution, displacement and thermal stresses are obtained may be applied to the design of useful structures or machines in engineering applications.

In chapter 7, the temperature distribution, displacement and thermal stresses at any point of a three dimensional rectangular plate have been obtained, when the boundary conditions are known, with the aid of finite Marchi-Fasulo transform technique. The results are obtained in the form of infinite series. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions. The temperature, displacement and thermal stresses are obtained may be applied to the design of useful structures or machines in engineering applications.

In chapter 8,the temperature distribution, displacement function and thermal stresses at any point of the cylinder have been derived, when the boundary conditions are known, with the help of finite Fourier sine transform, Laplace transform and Marchi-Zgrablich transform techniques. The expression (9.1) is represented graphically. The expressions are obtained in the form of infinite series. The results presented here will be more useful in Engineering problems particularly in the determination of the state of strain in cylinder constituting the foundations of container for hot gases or liquid in foundations for furnaces etc.

In chapter 9,, we discussed completely steady state problem of thermoelastic deformation of hollow cylinder with the known boundary conditions. The finite Hankel transform technique has been used to obtain the numerical results. The temperature displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

The equation (6.1) is represented graphically. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expression.

In chapter 10, the temperature, displacement and stress functions have been determined on upper plane surface of a finite length hollow cylinder with the help of finite Hankel transform technique. The results are obtained in terms of Bessel's function in the form of infinite series and that can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

In chapter 11, we discussed completely the inverse transient thermoelastic problem of a finite length hollow cylinder on outer curved surface, where homogeneous third kind boundary condition is maintained on the plane surfaces of the cylinder and on inner curved surface, the temperature is maintained at $u(z,t)$, which is a known function of (z,t). The finite Hankel transform and Laplace transform techniques are used to obtain the numerical results. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

In chapter 12, the displacement profile and thermal stresses of an isotropic hollow sphere have been derived. Modified Marchi-Zgrablich integral transform and Laplace transform are utilized to obtain the desired results. The expressions are represented graphically.

In chapter 13, by using the Laplace integral transform and the finite Marchi–Zgrablich integral transforms, the solutions for displacement, stress component and magnetic field vector perturbation for the FGM hollow sphere were obtained. The inhomogeneous constants presented in the present study are useful parameters from a design point of view. These constants can be varied for manufacturing and other specific applications and to control the distribution of magnetothermoelastic stresses and perturbation of magnetic field vector.

Chapter 14 deals with experiment carried out to study thermal behaviour of polymer substance PMMA(Polymethyl Methacrylate).Polymer material was analysed with the help of thermal mechanical analyser (TMA). This study helps to identify the thermal expansion coefficient (CTE) of PMMA and softening temperature. The softening temperature of the synthesized material was observed to be around 62 ºC. The observed CTE is -3.6×10^{-4} / ºC for the temperature range 65 to 75 deg. Thus, this study reveals the temperature sustainability of the material at room temperature and its usability up to 60^0C.