Bajaj College of Science, Wardha

Practice Sheet during lockdown

B.Sc. Sem VI

Subject : Mathematics Paper I

(Abstract Algebra)

(For any queries contact us on phone, whatsapp, email)

UNIT-I

1. If G is a group, then prove that A (G), the set of automorphism of G is also a group under composite composition of functions.

2. Are the following mappings $T : G \to G$ automorphisms of their respective groups.

   (i) G group of integers under addition, $T(x) = -x$

   (ii) G group of positive reals under multipliation, $T(x) = x^2$

3. Let G be a group; For $g \in G$ define $T_g: G \to G$ by $T_g x = g^{-1}xg, \forall x \in G$. Prove that T_g is an automorphism of G.

4. Prove that $I(G) \approx G/Z$, where I(G) is the group of inner automorphisms of G, and Z is the centre of G.

5. Let G be a group and ϕ an automorphism of G. If $a \in G$ is of finite order $o(a) > 0$, then prove that

$$o(\phi(a)) = o(a).$$

6. Show that in a group G, the mapping $T: G \to G$, given by $T(x) = x^{-1}\ \forall x \in G$ is an automorphism of G iff G is abelian. If $x_0 \in G$ with $x_0 \neq {x_0}^{-1}$, then $T \neq I$.

7. . Prove that Conjugacy is an equivalence relation on G.

8. Prove that N(a) is a subgroup of G.

9. If G is a finite group, then prove that $C_a = \frac{o(G)}{o(N(a))}$.

10. Let *Z* be an additive group of integers and $f: Z \to Z$ such that $f(x) = 2x, \forall x \in Z$. Show that f is an isomorphism but not an automorphismof Z.

UNIT-II

1. If R is a ring then for all $a, b \in \mathrm{R}$, prove that $(-\mathrm{a})(-\mathrm{b}) = \mathrm{ab}$.

2. Show that the commutative ring D is an integral domain iff for $a, b, c \in \mathrm{D}$ with $\mathrm{a} \neq 0$the relation $ab = \quad ac$ implies $b = c$

3. Prove that a finite integral domain is a field.

4. Prove that a homomorphism ∅ of a ring *R* into a ring *R'* is an isomorphism iff $Ker\ \emptyset = \{0\}$.

5. If ∅ is a homomorphism of R into R' with kernel $I(\emptyset)$, then Prove that (i) $I(\emptyset)$ is a subgroup of R under addition (ii) If $a \in I(\emptyset)$ and $r \in R$ then both ar and ra in $I(\emptyset)$.

6. If F is a field then prove that its only ideals are (0) and F itself.

7. The homomorphism ∅ of R into R' is an isomorphism iff $I(\emptyset) = \{0\}$.

8. Prove that the intersection of two left ideals of R is again a left ideal of R.

9. If R is a ring and for all $a, b \in R$, prove that $a(-b) = -(ab)$

10. If U is an ideal of R and $1 \in U$ then prove that $U = R$.

UNIT-III

1. In any vector space V, prove that (a) $(-1)\mathrm{u} = -\mathrm{u}$ for every $\mathrm{u} \in \mathrm{V}$.

   (b) $(-\alpha)\mathrm{u} = -(\alpha\mathrm{u})$ for every scalar α and for every $\mathrm{u} \in \mathrm{V}$.

2. A nonempty subset S of a vector space V is a subspace of V iff the following conditions are satisfied :

   (i) If $\mathrm{u}, \mathrm{v} \in \mathrm{S}$, then $\mathrm{u} + \mathrm{v} \in \mathrm{S}$.

   (ii) If $\mathrm{u} \in \mathrm{S}$ and α a scalar, then $\alpha\mathrm{u} \in \mathrm{S}$.

3. Prove that the intersection of two subspaces of a vector space is a subspace.

4. Let U and W are subspaces of a vector space V. Prove that $\mathrm{U} \cup \mathrm{W}$ is a subspace of V iff $\mathrm{U} \subset \mathrm{W}$ or $\mathrm{W} \subset \mathrm{U}$.

5. Define subspace of a vector space V. Prove that if S is a nonempty subset of a vector space V then L(S) is the smallest subspace of V containing S.

6. Define Linear span of a subset. Prove that if *S is a* subset of a vector space *V* then $L(S) = S \Leftrightarrow S$ is a subspace of V.

7. Let $v_1, v_2, \dots, v_n$ be n vectors of a vectors space V, prove that

(a) $[v_1, v_2, \dots, v_n] = [\alpha_1 v_1, \alpha_2 v_2, \dots, \alpha_n v_n]$ $\alpha_i \neq 0$ are scalars, $1 \leq i \leq n$.

(b) $[v_1, v_2] = [v_1 - v_2, v_1 + v_2]$

8. In the complex vector space V_2^c show that $(1 + i, 1 - i) \in [(1 + i, 1), (1, 1 - i)]$.

9. Let V be any vector space and $v, v_1, v_2, \dots, v_n \in V$.

(a) If v is a linear combination of $v_1, v_2, \dots, v_n$ i.e $v \in [v_1, v_2, \dots, v_n]$, then $\{v, v_1, v_2, \dots, v_n\}$ is LD.

(b) If $\{v_1, v_2, \dots, v_n\}$ is LI and $v \notin [v_1, v_2, \dots, v_n]$, then $\{v, v_1, v_2, \dots, v_n\}$ is LI.

10. Show that the ordered set $\{(1,1,0), (0,1,1), (1,0,-1), (1,1,1)\}$ is L.D.

Find the largest L.I. subset of S whose span is [S].

11. If u, v and w are three linearly independent vectors of a vector space V, then prove that

$u + v,\ v + w$ and $w + u$ are also L.I.

12. Define basis of a vector space. Prove that in an n-dimensional vector space V, any set of n- linearly independent vectors is a basis.

13. Prove that the set *B* ={(1,0,0), (1,1,0), (1,1,1)} form a basis for V_3.

14. Prove that the set $\{(1,1,1), (1,-1,1), (0,1,1)\}$ is a basis for V_3.

15. Prove that the vectors (1,0,1) ,(1,1,0) and (1,1,-1) are LI.

UNIT-IV

1. Prove that $T : V_3 \to V_2$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2,\ x_1 + x_3)$ is a linear transformation.

2. Show that a mapping $T : V_2 \to V_2$ be defined by

$T(x,\ y) = (x \cos\theta - y \sin\theta,\ x \sin\theta + y \cos\theta$ is linear.

3. Let U and V be vector spaces over the field F and $T : U \to V$ be a linear map. Then

$(a)\quad T(0)=0$ $\qquad$ $(b)\quad T(-u)=-T(u)\,,\quad \forall\ \ u\in U$

$(c)\ \ T(\alpha_1 u_1+\alpha_2 u_2+\cdots+\alpha_n u_n)=\alpha_1\,T(u_1)+\alpha_2\,T(u_2)+\cdots+\alpha_n\,T(u_n)\,,$

$\forall\ \ u_i\in U\ \ \&\ \ \alpha_i\in F\,,\ 1\le i\le n\ ,\ \ n\in N.$

4. Let $T:\ U\to V$ be a linear map. Then prove that

(a) $R(T)$ is a subspace of V $\qquad$ (b) $N(T)$ is a subspace of U.

(c) T is one-one iff $N(T)$ is the zero subspace of U.

(d) If $[\,u_1,\ u_2,\ \dots\ ,u_n]=U,$ then $R(T)=[\,T(u_1),\ T(u_2),\ \dots\ ,T(u_n)\,]$.

5. State and prove Rank-Nullity Theorem

6. Determine range, rank, kernel and nullity of the linear map $T:\ V_3\to V_4$ defined by

$T(x_1,x_2,x_3)=(x_1,\ x_1+x_2,\ x_1+x_2+x_3,\ x_3)$. Is T one-one?

7. Let $T:V_4\to V_3$ be a linear map defined by

$T(e_1)=(1,1,1),\ T(e_2)=(1,-1,1), T(e_3)=(1,0,0),\ T(e_4)=(1,0,1)$. Verify Rank-nullity theorem.

8. If U and V are finite dimensional vector spaces of same dimension, then a linear map $T:U\to V$ is one-one iff T is onto.

9. Prove that the linear map $T:V_3\to V_3$ defined by

$T(e_1)=e_1+e_2$, $T(e_2)=e_2+e_3$, $T(e_3)=e_1+e_2+e_3$ is non singular and find its inverse.

10. Let $T:V_3\to V_3$ be a linear map defined by

$T(e_1)=e_3$, $T(e_2)=e_1$, $T(e_3)=e_2$. Prove that $T^2=T^{-1}$.

11. Show that the linear map $T:V_2\to V_2$ defined by

$T(x_1, x_2) = (\alpha_1 x_1, \ \alpha_2 x_2), \quad \alpha_1 \neq 0, \ \alpha_2 \neq 0$ is non singular and find its inverse.

12. Find the matrix of linear transformation $T : V_3 \to V_3$ defined by

$T(x, y, z) = (x - y + z, \ 2x + 3y - \frac{z}{2}, \ x + y - 2z)$ relative to the ordered bases

$B_1 = \{(-1,1,0), (5,-1,2), (1,2,1)\}$ and $B_2 = \{(1,1,0), (0,0,1), (1,5,2)\}$.

13. Find the matrix of linear transformation $T : V_3 \to V_3$ defined by

$T(x_1, x_2, x_3) = (x_1 - x_2 + x_3, \ 2x_1 + 3x_2 - \frac{1}{2}x_3, \ x_1 + x_2 - 2x_3)$ relative to the bases $B_1 = \{e_1, e_2, e_3\}$ and $B_2 = \{(1,1,0), (1,2,3), (-1,0,1)\}$.

14. Find the matrix of linear transformation for the linear map $T : V_3 \to V_3$ defined by

$T(x, y, z) = (z, \ y + z, \ x + y + z)$ relative to the basis $\{(1,0,1), \ (-1,2,1), \ (2,1,1)\}$.

15. If matrix of linear map T with respect to the bases B_1 and B_2 is $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

where $B_1 = \{(1,2,0), (0,-1,0), (1,-1,1)\}$ and $B_2 = \{(1,0), (2,-1)\}$. Find $T(x, y, z)$.

16. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a matrix of linear map T with respect to the bases $B_1 = \{(1,1,1), (1,0,0), (0,1,0)\}$ and $B_2 = \{(1,2,3), (1,-1,1), (2,1,1)\}$. Find $T(x, y, z)$.

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B.Sc. Final Sem VI

Subject : Mathematics Paper II

(Special Theory of Relativity)

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UNIT I

1. Prove that in an inertial frame a body not under the influence of any forces, moves in a straight line with constant velocity.

2. Derive Galilean transformations

3. Show that acceleration is invariant under G. T.

4. State and prove the principle of classical or Newtonian relativity.

5. Show that the Newton's Kinematical equations of motion are invariant under Galilean transformations.

6. Prove that the Maxwell's equations are not valid in all inertial co-ordinate systems under G.T.

7. Describe the Michelson–Morley experiment. Explain the null result w r t ether.

8. In an experiment, the length of the arm of the interferometer was 11 meters, the wave length of light 5.5×10^{-5} cm and the earth's velocity $30\ km/sec$, calculate the amount of fringe-shift.

9. State the Fundamental postulates of Einstein's special theory of Relativity

10. The space- time co-ordinates of two events as measured in an inertial frame $\boldsymbol{S}$ are $(\boldsymbol{x_1}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{t_1})$ and $(\boldsymbol{x_2}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{t_2})$. Show that there exists an inertial frame moving with

uniform velocity $\boldsymbol{c^2(t_2 - t_1)/(x_2 - x_1)}$ with respect to $\boldsymbol{S}$ and where these events occur at the same time $\boldsymbol{(x_2t_1 - x_1t_2)/\sqrt{(x_2 - x_1)^2 - c^2(t_2 - t_1)^2}}$ ..

11. One event occurs at the origin of the inertial frame $\boldsymbol{S}$ at $\boldsymbol{t = 0}$. A second event occurs at the point $\boldsymbol{x = 4c}$, $\boldsymbol{y = 0}$, $\boldsymbol{z = 0}$ at time $\boldsymbol{t = 2}$ second relative to $\boldsymbol{S}$. Find the velocities relative to $\boldsymbol{S}$, of the inertial frames in which

(a) the events are simultaneous

(b) event 2 precedes event 1 by 2 seconds.

(c) events are recorded at the same point.

UNIT II

1. Obtain transformation of particle velocities.

2. Obtain the relativistic addition law for velocities.

3. Obtain the transformations for the acceleration of a particle.

4. Obtain the transformation of the Lorentz contraction factor $\boldsymbol{(1 - \frac{u^2}{c^2})^{1/2}}$.

5. Show that the simultaneity is relative in SR.

6. Explain the phenomenon of time dilation in SR.

7. Explain the length contraction in SR.

8. A clock keeps correct time, with what speed should it be moved relative to an observer so that it may seem to loose 1 minute in 24 hours.

9. What will be the apparent length of meter stick measured by an observer at rest when the stick is moving along its length with a velocity equal to $c, \frac{c}{\sqrt{2}}$, $0.8\,c$, $o.5\,c$.

10. Calculate the percentage contraction of a rod moving with velocity 0.8 c along its own length.

UNIT III

1. Obtain the components of a contravariant vector in the x'-coordinate system if $x'^1 = 7\,(x^1)^2,\; x'^2 = 6(x^1)^2 + 2\,(x^2)^2$ and its components in x-coordinate system are 6 and 3.

2. Find the covariant components of a tensor in cylindrical coordinates $\rho,\ \phi,\ z$ if its covariant components in rectangular coordinates $x,\ y,\ z$ are $2x - z,\ x^2y$ and yz.

3. Prove that the Kronecker delta δ_b^a is a mixed tensor of rank two.

4. Show that any tensor of second order (covariant/contravariant) may be expressed as the sum of symmetric and skew-symmetric tensors.

5. Show that $a_{mn}\, x^m\, x^n = 0$ for a skew-symmetric tensor a_{mn}.

6. Determine the metric and conjugate metric tensor in cylindrical coordinates.

7. Find (a) g and (b) g^{jk} corresponding to

$$ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6\, dx^1 dx^2 + 4\, dx^2 dx^3.$$

8. There exists an inertial system S' in which the two events occur at one and the same point if the interval between two events is time-like.

9. There exists an inertial system S' in which the two events occur at the same time if the interval between two events is space-like.

10. Define four tensor . Obtain transformation for T^{14}

UNIT - IV

1. Derive the formula $E = mc^2$.

2. Derive transformation formula for momentum and energy

3. Derive transformation formula for mass of a particle

4. Obtain the wave equation for the propagation of electric field strength E and magnetic field strength H.

5. Derive maxwell's qquations in index form

6. Show that the four velocity and four acceleration are mutually orthogonal.

7. Obtain the expression relativistic mass $m = m_0^2 \ / \sqrt{1 - u^2 / c^2}$.

8. A particle is given kinetic energy equal to *n* times its rest energy $m_0 c^2$ what is its speed

9. Define four velocity and four acceleration

Stay home, Stay safe. Best wishes.