Bajaj College of Science, Wardha

Practice Sheet during lockdown

B.Sc. Sem IV

Subject : Mathematics Paper I

Partial Differential Equations & Calculus of Variations

(For any queries contact us on phone, whatsapp, email)

UNIT-I

1. Form the PDE by eliminating the arbitrary Function from the equation

   (i) $f(x^2+y^2+z^2, z^2-2xy)=0$ (ii) $z=xy+f(x^2+y^2)=0$ (iii) . $z=(x+a)(y+b)$

2. Find the general integral of the PDE $z(xp-yq)=y^2-x^2$

3. Find the general integral of the PDE $(y+zx)p-(x+yz)q)=x^2-y^2$

4. Find the integral surface of the PDE $2y(z-3)p+(2x-z)q=y(2x-3)$ which passes through $x^2+y^2=1$, $z=1$

5. Find the integral surface of the PDE $(x-y)p+(y-x-z)q=z$ which passes through the circle $x^2+y^2=2x$, $z=0$

6. Solve the PDE $p^2+q^2=x^2+y^2$

7. Solve the PDE $p=(z+qy)^2$ by Charpit's method

8. Solve the PDE $z^2=pqxy$ by Charpit's method

9. Solve the PDE $(p^2+q^2)y=qz$ by Charpit's method

10. Solve the PDE $xpq+yq^2=1$ by Jacobi's method

11. Solve the PDE $p^2x+q^2y=z$ y by Jacobi's method

UNIT-II

1. Prove that $\frac{1}{D-mD'} f(x,y)=\int f(x, c-mx)\, dx$, $y=c-mx$.

2. Solve $\left(D^2+DD'-6D'^2\right)z=\cos(2x+y)$

3. Solve$\left(D^2 - 3DD' + 2\,D'^2 \right) z \;=\; e^{2x+3y} + \sin(x - 2y)$

4. Solve $\left(D^2 - 6DD' + 9\,D'^2 \right) z \;=\; 12\,x^2 + 36xy$

5. Solve $\left(D^2 + 3DD' + 2\,D'^2 \right) z \;=\; x + y$

6. Solve $\left(D^2 + DD' - 6\,D'^2 \right) z \;=\; y \cos x$

7. Solve$\left(D^2 - 2DD' - 8\,D'^2 \right) z \;=\; \sqrt{2x + 3y}$

8. Solve$\left(D^2 - 2DD' + D'^2 \right) z \;= \tan(x + y)$

9. Solve $4r - 4s + t = 16\,\log(\,x + 2y)$

10. Solve$\left(D^2 -\; D\,D' +\; D'^2\right) z \;= e^{x+2y} +$
x^3

11. Solve $\frac{\partial^2 z}{\partial x^2} + 3\,\frac{\partial^2 z}{\partial x\,\partial y} + 2\,\frac{\partial^2 z}{\partial y^2} \;= 12xy$

12. Prove that The solution (CF) of a non-homogeneous PDE $(D - mD' - \alpha)z = 0$ is

$$z \;=\; e^{\alpha x}\,\emptyset(y + mx).$$

13. Solve $(\,D^2 - DD' +\; D' - 1\,)\,z \;=\; e^{y} + \cos(x + 2y)$

14. Solve $(D - D' - 1)(D + D' - 3)(D + D')z \;=\; e^{x+y-3}\,\sin(2x - y)$

15. Solve $x^2\,\frac{\partial^2 z}{\partial x^2} -\; y^2\,\frac{\partial^2 z}{\partial y^2} \;= x^2 y$

UNIT-III

1. Solve $yt - q = xy$ by reducing to linear form.

2. Solve $s - t = \frac{x}{y^2}$

3. Solve $xr + p = 9x^2y^3$

4. Solve $t + s + q = 0$

5. Solve $xys - qy = x^2$

6. Find the characteristics of $y^2 r - x^2 t = 0$

7. Find the characteristics of $x^2 r + 2xys + y^2 t = 0$

8. Reduce the equation $r = x^2 t$ to canonical form

9. Reduce the equation $r = (1+y)^2 t$ to canonical form

10. Reduce the equation $r + 2xs + x^2 t = 0$ to canonical form

11. Classify the partial differential equation $2u_{xx} + 4u_{xy} + 3u_{yy} = 2$

UNIT-IV

1. Find the distance between the curves $y_1(x) = xe^{-x}$, $y_2(x) = 0$ on $[0, 2]$.

2 Prove that a necessary condition for the functional $I[y(x)] = \int_{x_0}^{x_1} F(x, y, y')dx$

to be an extremum is that $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y\prime}\right) = 0$

3. If the function F depends on y' alone then prove that the extremals are all straight lines.

4. Test for an extremum the functional

$$I[y(x)] = \int_0^1 (y'^2 + y' + 1)\, dx \quad ; \quad y(0) = 1\,, \; y(1) = 2$$

5. On which curve the functional

$$I[y(x)] = \int_0^{\pi} (y'^2 - y^2 + 4y\cos x)\, dx \;, \qquad y(0) = 0\,, \; y(\pi) = 0 \;\; \text{be extremized ?}$$

6. Find the curve on which the functional

$$I[y(x)] = \int_1^2 \frac{x^3}{y'^2}\, dx \qquad \text{with} \quad y(1) = 0 \;\text{ and }\; y(2) = 3 \quad \text{can be extremized ?}$$

7. Find the extremal of the functional

$$I[y(x)] = \int_{-1}^{0} (12xy - y'^2)\, dx \quad \text{that satisfies the boundary conditions } y(-1) = 1, y(0) = 0.$$

8. Find the extremal of the functional

$$I[y(x)] = \int_{0}^{2\pi} (y'^2 - y^2)\, dx \quad \text{that satisfies the boundary conditions } y(0) = 1, y(2\pi) = 1$$

9. Prove that the shortest distance between two points in a plane is a straight line.

10. Find the extremal of the functional $I[y(x)] = \int_{-1}^{0} (480y + y'''^2)\, dx$;

$$y(0) = y'(0) = y''(0) = 0, \qquad y(-1) = \frac{1}{3}, \qquad y'(-1) = -2, \qquad y''(-1) = 8.$$

11. Find the extremal of the functional

$$I = \int_{0}^{\pi} (y'^2 - 2y^2 + 2yz - z'^2)\, dx \quad ; \quad y(0) = z(0) = 0, \quad y(\pi) = z(\pi) = 1.$$

12. Write Euler's-Ostrogradsky equation for the functional

$$I[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 \right] dx\, dy$$

13. Find the extremal of the functional

$$I[y(x)] = \int_0^{\pi} (16\, y^2 - y''^2 + x^2)\, dx \, ; \; y(0) = y(\pi) = 0, y'(0) = y'(\pi) = 1$$

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B.Sc. Sem IV

Subject : Mathematics Paper II

Mechanics

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UNIT I

1. Show that a system of coplanar forces acting at different points of a rigid body can be reduced to a single force through a given point and a couple.
2. Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A . Show that there is a thrust in BD equal to $\frac{W}{\sqrt{3}}$.
3. The moments of a given system of forces about three points (2,0),(0,2) and (2,2) are 3,4 and 10 respectively. Find the magnitude of the resultant force and equation of its line of action
4. Derive the intrinsic equation of a common catenary in the form $s = c\ tan\psi$
5. A uniform chain of length l is suspended from two points in the same horizontal line such that the maximum tension is equal to n times its weight, show that the least possible sag in the middle is $l\left[n - \sqrt{n^2 - \frac{1}{4}}\right]$.
6. Derive the relation $y^2 = s^2 + c^2$ for a common catenary

UNIT II

1. Prove that $\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{n}$, $\frac{d\,\hat{n}}{dt} = -\left(\frac{d\theta}{dt}\right)\hat{r}$
2. Derive relation for radial velocity $v_r = \dot{r}$

3.The velocities of a particle along and perpendicular to a radius vector from a fixed origin are λr^2 and $\mu\theta^2$. Show that the equation of the path is $\frac{\lambda}{\theta} = \frac{\mu}{2r^2} + c$

4. The velocities of a particle along and perpendicular to the radius vector are λr and $\mu\theta$. Find the path and show that the acceleration along and perpendicular to the radius vector are $\lambda^2 r - \frac{\mu^2 \theta^2}{r}$ and $\mu\theta\,(\lambda + \frac{\mu}{r})$ respectively.

5. An insect crawls at a constant rate u along the spoke of a cart wheel of radius a , the cart moving with velocity v . Find the accelerations along and perpendicular to the spoke.

6. A particle describes the cycloid $s = 4\,a \sin\psi$ with uniform speed v . Find its acceleration at any point.

7. Derive the relation $\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{n}$

8. Define period and Frequency of SHM

UNIT III

1. Derive Lagranges equations in polar coordinates:

$$m\ddot{r} - mr\dot{\theta}^2 = F_r\ , \qquad mr\,\ddot{\theta} + 2\,m\dot{r}\dot{\theta} = F_\theta$$

2. (Conservation theorem for the linear momentum of a system of particles): If the total external force on the system of particles is zero, the total linear momentum is conserved.

3. State and prove . D’Alembert’s Principle:

4. Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$ then show that the common acceleration of the particle is $\frac{m_1 - m_2}{m_1 + m_2}\,g$

5. The centre of mass of the system of particles moves as if the total external force were acting on the entire mass of the system concentrated at the centre of mass.

6. Construct the Lagrangian for a particle moving in space and then deduce the equations of motion.

<u>UNIT - IV</u>

1. Show that the problem of motion of two masses interacting only with one another can always be reduced to a problem of motion of a single mass.
2. If the potential energy is a homogeneous function of degree -1 in the radius vector $\overline{r_i}$ then show that the motion of a conservative system takes place in a finite region of space only if the total energy is negative
3. Define velocity dependent potential
4. If $F = -c/r^3$, c = constant, then find V
5. Show that in a central force field the angular momentum L of a particle remains constant.
6. Show that the path of a particle in a central force field lies in one plane
7. Show that for a central force field F the differential equation for the orbit is given by $\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2u^2}F\left(\frac{1}{u}\right),\ u = \frac{1}{r}$
8. Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle, then the force varies as the inverse fifth power of the distance.
9. Find the law of force if a particle moves along a curve $r^n a^n cos n\theta$ under the influence of a central force.

Stay home, Stay safe. Best wishes.