Bajaj College of Science, Wardha

Practice Sheet during lockdown

B.Sc. Sem II

Subject : Mathematics Paper I

(Differential, Difference Equations & Solid Geometry)

(For any queries contact us on phone, whatsapp, email)

UNIT-I

1. Solve $x\frac{dy}{dx} + y = x\ \log x$

2. Solve $x\frac{dy}{dx} + y = x^3\ y^6$

3. Solve $\frac{dy}{dx} = y\tan x - 2\ sinx$

4. Solve $\frac{dy}{dx} + y\ tanx = x^2\ cos^2x$

5. Show that the following DE is exact and solve it.

$$\left(1 + e^{x/y}\right)dx + e^{x/y}\ (1 - \frac{x}{y}\ )dy = 0$$

6. Solve $\frac{1}{y}\frac{dy}{dx} + \frac{x}{1-x^2} = xy^{-1/2}$

7. Solve $y - 2px =\ tan^{-1}(xp^2)$

8. Solve $p^2 - 4p + 3 =\ 0$

9. Solve $p^3 - 2xyp + 4y^2 =\ 0$

10. Solve $p = \sin\,(y - xp)$

UNIT-II

1. Solve $(D^2 - 3D + 2)y = x\,e^{3x}$

2. Solve $(D^2 - 5D + 6)y = \cos 3x$

3. Solve $(D^2 - 4D + 3)y = 2x^2$

4. Solve $(D^2 - D)y = x^2 + 3x + 5$

5. Solve $(D^2 + n^2)y = cosec\,nx$

6. Solve $(D^2 + 1)y = sec^2x$

7. Solve $x^2\frac{d^2y}{dx} - 8x\frac{dy}{dx} + 8y = logx$

8. Solve $x^2\frac{d^2y}{dx} - 3x\frac{dy}{dx} + 4y = 2x^2$

9. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by method of variation of parameters.

10. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2\,e^x$ by method of variation of parameters.

UNIT-III

1. From the equation $y_n = A2^n + B(-3)^n$, derive a difference equation by eliminating the arbitrary constants A

and B. What is the order of the difference equation?

2. Find the difference equations from the equation $u_x = A\cos x\theta + B\sin x\theta$

3. Solve $u_{x+1} - 3^x u_x = 0$

4. Solve $y_{x+2} - 2y_{x+1} + y_x = 2^x x^2$

5. Solve $u_{x+2} + u_{x+1} + u_x = x^2 + x + 1$.

6. Solve $y_{n+2} - 4y_n = 9n^2$

7. Solve $u_{x+2} - 7u_{x+1} + 10u_x = 12(4)^x$

8. Solve $y_{n+2} - y_{n+1} - 12y_n = \cos 2n$

9. Solve $y_{n+2} - 7y_{n+1} + 12y_n = \cos n$

10. Define difference equation and its order

UNIT-IV

1. Find the equation of the sphere which passes through the point (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its centre on the plane $x + y + z = 6$.

2. Prove that the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ will intersect the plane $l x + m y + n z = P$ if and only if $(l u + m v + n w + p)^2 \leq (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$

3. Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ is a great circle.

4. Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle.

5. Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at $(1, 2, -2)$ and passes through the point $(1, -1, 0)$.

6. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact.

7. Find the equations of the spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch $4x + 3y - 15 = 0$.

8. Show that the plane $2x - 2y + z + 16 = 0$ touches the sphere $x^2 + y^2 + z^2 + 2x - 4y + 2z = 3$ and find the point of contact.

9. Define right circular cylinder and right circular cone.

10. Find the equation of the sphere with centre (3, - 4, 5) and radius 5

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UNIT I

UNIT I

1. A particle moves along the curve $\bar{r} = (t^3 - 4t)\,\bar{\imath} + (t^2 + 4t)\,\bar{\jmath} + (8t^2 - 3t^3)\,\bar{k}$ where t is the time. Find the magnitude of the tangential components of its acceleration at $t = 2$.
2. A particle moves so that its position vector is given by $\bar{r} = cos\omega t\,\bar{\imath} + sin\omega t\,\bar{\jmath}$ where ω is a constant. Show that (a) the velocity $\bar{v}$ of the particle is perpendicular to $\bar{r}$, (b) the acceleration $\bar{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin and (c) $\bar{r} \times \bar{v} = \bar{a}$ constant vector.
3. A particle moves along a curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\bar{\imath} + \bar{\jmath} + 3\,\bar{k}$.
4. Find $\nabla\emptyset$ if $\emptyset = \ln|\,\bar{r}\,|$ (b) $\emptyset = \frac{1}{r}$ (c) $\emptyset = r^n$
5. Prove that $r^n\,\bar{r}$ is $irrotational$. Find the value of n when it is $solenoidal$.
6. Find the directional derivative of the function $\emptyset = x^2 - y^2 + 2\,z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$.
7. Prove that for any vector field $\bar{V}$, curl (curl $\bar{V}$) = 0 vector
8. If $\bar{A}$ is a constant vector, prove that $\nabla(\,\bar{r}\cdot\bar{A}) = \bar{A}$.
9. Find a unit normal vector to the surface $x^2 + y^2 = z^2$ at the point $(1, 0, -1)$.
10. Prove that $i)$ $\nabla \times (\nabla\emptyset) = 0$ $ii)$ $\nabla \cdot (\nabla \times \bar{A}) = 0$
11. If $\bar{v} = \bar{w} \times \bar{r}$, prove that $\bar{w} = \frac{1}{2}\,curl\,\bar{v}$, where $\bar{w}$ is a constant vector.

12. If $\bar{F} = 3xy\ \bar{\imath} - y^2\,\bar{\jmath}$, evaluate $\int_C\ \bar{F}.d\bar{r}$ where C is the curve in the xy plane, $y = 2x^2$ from $(0,0,)$ to $(1,2)$.

13. Find the total work done in moving a particle in a force field given by $\bar{F} = 3xy\,\bar{\imath} - 5z\,\bar{\jmath} + 10x\,\bar{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

14. Find the work done in moving a particle once around a circle C in the xy plane of radius 2 and centre $(0,0)$ and if the force field is given by $\bar{F} = 3xy\,\bar{\imath} - y\,\bar{\jmath} + 2xz\,\bar{k}$.

15. Find the work done in moving a particle once around a circle C in the xy plane. If the circle has centre at the origin and radius 3 and the force field is given by

16. Find the work done by a force $y\,\bar{\imath} + x\bar{\jmath}$ which displace a particle from origin to a point $(\bar{\imath} + \bar{\jmath})$.

UNIT II

1. Evaluate $\iint_R xy\ dx\,dy$, where R is the quadrant of the circle $x^2 + y^2 = a^2$ where $x \geq 0$, $y \geq 0$.

2. Evaluate $\iint_R x^2\ dx\,dy$, where R is the region bounded by the curves $y = x$ and $y = x^2$.

3. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\ dy\,dx$ by changing the order of integration.

4. Change the order of integration in the following integral $\int_0^a \int_0^x f(x,y)\,dx\,dy$

5. Evaluate $\int_0^a \int_y^a \frac{x}{(x^2+y^2)}\ dx\,dy$ by changing in to polar coordinates.

6. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\ dydxdz$

7. Evaluate $\iiint_S\ dx\,dy\,dz$, where

$$S = \{(x,y,z):\ x^2 + y^2 + z^2 \leq 1,\ x \geq 0,\ y \geq 0,\ z \geq 0\}.$$

8. Evaluate $\int_0^{2\pi} \int_0^{\pi/4} \int_0^a r^2 \sin\theta\, dr\, d\theta\, d\emptyset$.

9. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x^2-y^2} dz\,dy\,dx$.

UNIT III

10. Apply Green's theorem to prove that

a) The area enclosed by a simple plane curve C is $= \frac{1}{2} \oint_C (xdy - ydx)$

b) Hence find the area of an ellipse whose semi –major and semi-minor axes are of length a and b.

11. Show that $\iint_S \bar{r} \cdot \hat{n}\, dS = 3V$, where V is the volume enclosed by a closed surface S.

12. If $\bar{H} = curl\, \bar{A}$ then by divergence theorem, prove that $\iint_S \bar{H} \cdot \bar{n}\, dS = 0$, for any closed surface S.

13. Show that $\iint_S \left(ax\bar{\imath} + by\bar{\jmath} + cz\bar{k}\right) \cdot \hat{n}\, dS = \frac{4}{3}\pi(a + b + c)$,

14. Evaluate $\oint_C (y - \sin x)dx + \cos x\, dy$ by using Green's theorem in a plane, where C is the triangle in xy-plane with vertices $(0,0)$, $(\pi/2, 0)$, $(\pi/2, 1)$.

Example 2. Evaluate by Stoke's theorem $\int_C (e^x dx + 2ydy - dz)$, where C is the curve $x^2 + y^2 = 4, \quad z = 2$.

15. Evaluate using divergence theorem for $\bar{f} = (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k}$ taken over the rectangular parallelepiped $0 \le x \le a, \ 0 \le y \le b, \ 0 \le z \le c$.

UNIT IV

1. Test the convergence of $\int_1^\infty \frac{xdx}{2x^4 + 3x^2 + 5}$

2. Prove that $\int_0^\infty \frac{\cos x}{x^2 + 1} dx$ is convergent.

3. Test the convergence of $\int_1^5 \frac{dx}{\sqrt{x^4 - 1}}$.

4. Prove that: $\Gamma(n+1) = n\Gamma(n), n > 0$ and $\Gamma(n+1) = n!,\ n = 1, 2, 3 - - -$

5. Prove that: $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

6. Prove that: $\int_0^\infty x^{n-1} e^{-kx} dx = \frac{\Gamma(n)}{k^n}$.

7. Prove that: $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$.

8. Prove that : $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \dfrac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$ $m, n > 0$

9. Prove that: $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ $m,\ n > 0$.

10. Given that $\int_0^{\infty} \dfrac{x^{p-1}}{1+x} dx = \dfrac{\pi}{\sin p\pi}$,

11. Show that $\Gamma(p)\Gamma(1-p) = \dfrac{\pi}{\sin p\pi}$ where $0 < p < 1$.

12. Prove: $\beta(m,n) = \int_0^{\infty} \dfrac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^{\infty} \dfrac{x^{n-1}}{(1+x)^{m+n}} dx$.

13. Show that : $\beta(m,n) = \int_0^{1} \dfrac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$.

14. Evaluate $\int_0^{\frac{\pi}{2}} \cos^6 \theta \, d\theta$, $\int_0^1 \sqrt{x(1-x)dx}$

Stay home, Stay safe. Best wishes.