Chapter 14

MAGNETIC EFFECT OF ELECTRIC CURRENT

Ampere's Circuital Law:

Ampere's Circuital law is an alternative to Biot-Savert law. But, it is useful to calculating magnetic field only in situations with considerable symmetry.

Statement: "The line integral of magnetic field $\overline{B}$ around any closed path in vacuum or air is equal to μ_0 times the total current enclosed by that path".

Mathematically,

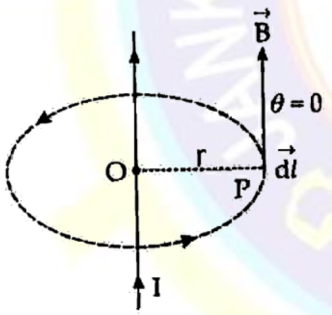
$$\oint \overline{B}.\overline{dl} = \mu_0 I$$

Where, $\oint \overline{B}.\overline{dl}$ = line integral of $\overline{B}$ around the closed path and

I = Current enclosed by that Path.

This results holds good irrespective of the size and shape of the closed path enclosing the current. This law is true for steady currents only.

Proof:



Consider, a long straight conductor carrying current (I) in the direction as shown in above fig. Due to current in the conductor, the magnetic lines of force are in the form of concentric circle such that conductor is at the centre of these circles. The magnitude of magnetic field at point (P), at perpendicular distance are from the conductor is given by

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{--------------- (1)}$$

Consider, a magnetic lines of force of radius (R) on this closed path, the magnitude of $\vec{B}$ and $\vec{dl}$ at every point on this path is –

$$\theta = 0^0$$

$$\therefore \quad \cos\theta = 1$$

$$\therefore \quad \oint \vec{B}.\vec{dl} = \oint Bdl\cos\theta$$

$$= \oint B.dl.1$$

$$= \oint B.dl.$$

$$\therefore \quad \oint \vec{B}.\vec{dl} = \oint Bdl\ .$$

But, $\oint \vec{dl}$ = Circumference of closed path.

$$= 2\pi\ r$$

$$\therefore \quad \oint \vec{B}.\vec{dl} = B \times 2\pi\ r$$

Using equation (1),

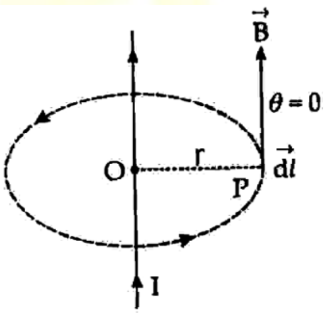
$$\oint \vec{B}.\vec{dl} = \frac{\mu_0 I}{2\pi\ r} \times 2\pi\ r$$

$$\therefore \quad \oint \vec{B}.\vec{dl} = \mu_0 I$$

Application of Ampere's Circuital Law:

Expression for magnetic field due to a straight conductor carrying current:

Consider, a long straight conductor carrying current (I) in the direction as shown in fig. We have to determine value of magnetic field $\vec{B}$ at a point P at perpendicular distance (r) from the conductor.



The Magnetic lines of force are in the form of concentric circles around the conductor. Hence $\vec{B}$ is directed tangentially at every point of the closed path. Such that the angle between $\vec{B}$ and $\vec{dl}$ is zero

i.e. $\theta = 0^0$.

$$\therefore \quad \cos\theta = 1$$

Now, applying Ampere's circuital law, to closed Path –

$$\oint \vec{B}.\vec{dl} = \oint B.dl\cos\theta$$

But, $\theta = 0^0$.

$\therefore$ $\cos\theta = 1$

$$\therefore \oint \vec{B}.\vec{dl} = \oint B.dl.$$

$$= B.\oint dl. = B \times 2\pi\, r \text{ ------(1)}$$

But, According to Ampere's Circuital law,

$$\oint \vec{B}.\vec{dl} = \mu_0 I \text{ ---------- (2)}$$

Equating (1) and (2),

$$\therefore \quad \mu_0 I = B \times 2\pi\, r$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi\, r} \text{ -------------- (3)}$$

This is the expression for magnetic field due to straight conductor carrying current.

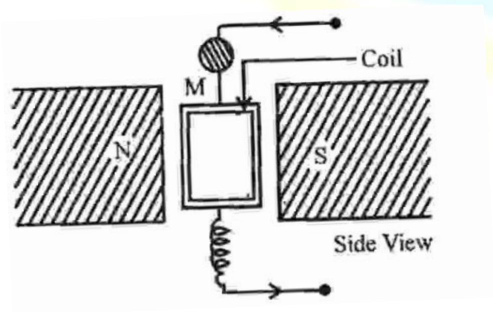
Principle, Construction, Theory and Working of Moving coil Galvanometer (M. C. G.):-

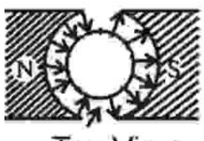
Principle: When a current carrying coil is suspended in uniform magnetic field, a torque acts on it, due to which coil deflects. The deflection produced in a coil in a M. C. G. is directly proportional to the current flowing through the coil.

If I is the current flowing through the coil and θ be the deflection produced in it, then –

$$I \propto \theta$$

Construction (Suspended type M. C. G.)





Top View

1. Horse Shoe Magnet: A strong Horseshoe magnet with concave pole pieces is used to produce radial magnetic field in M.C.G.

2. Coil: A rectangular coil of thin insulated copper wire is suspended between the poles of the Horseshoe magnet. It is free to rotate about vertical axis. The coil is suspended with the help of a thin phosphor Bronze wire. The lower end of the coil is connected to a spring. Current enters in the coil through the wire and leaves through the spring.

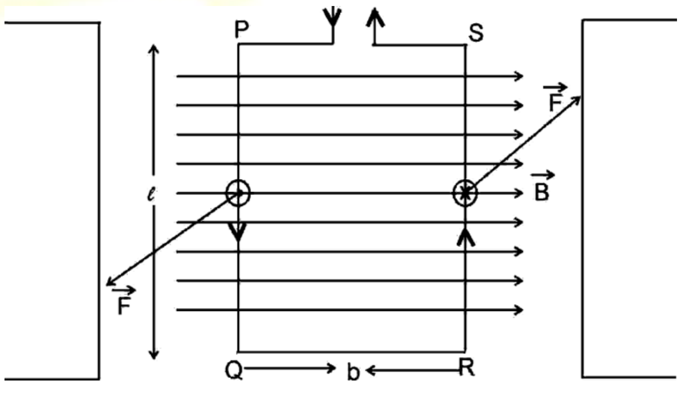
3. Iron Core: A cylinder made up a soft iron is fixed in the gap of the coil in such a way that the coil can freely rotate around it. As the permeability of soft iron is high, the iron core increases the strength of the radial magnetic field.

4. Lamp and Scale arrangement: A small mirror is mounted on the suspended wire. A lamp and a scale are arranged in front of the mirror. The reflected bright spot on the scale gives the measure of deflection of coil.

Theory and Working:

Consider a rectangular coil PQRS of length (l) and breadth (b) is suspended in uniform magnetic field of induction $\left(\vec{B}\right)$. Let, I be the current passing through it.

Let, the plane of the coil is parallel to the direction of magnetic field. In such a case, PS and QR are parallel to the field. The force acting on the sides is given by –



$$F = BI\, l\sin\theta$$

But, $\theta = 0^0$.

$\therefore$ $\sin\theta = 0$

$\therefore$ $F = 0$.

i.e. no force is experienced by PS & QR.

Sides PQ & SR are perpendicular to the direction of magnetic field.

$\therefore$ $\theta = 90^0$.

$\therefore$ $\sin\theta = 1$

$\therefore$ $F = B\,I\,l$.

i.e. the maximum force acts.

The direction of the force can be determined by Fleming's Left hand rule. It is observed that, this force acts normally outward on PQ while normally inward on SR. The two equal and opposite forces separated by perpendicular distance B forms a couple which is nothing but torque. Therefore,

Torque, $\tau = F \times b$

$\therefore$ $\tau = B\,I\,(l \times b)$

But, $l \times b = A$ = Area of the coil

$\therefore$ $\tau = BIA$.

If coil is having n number of turns, then –

$$\tau = n\,B\,I\,A$$

Due to this torque, coil deflects.

A radial magnetic field is used in M. C. G. Therefore, it remains parallel to the PS and QR in any position. Thus, torque (τ) remains uniform in any position of the coil.

Let coil come to the rest at an angle of twist (θ), then, the restoring torque is given by –

$$\tau' = C\,\theta$$

Where, C = Restoring torque per unit twist called tortional constant.

In the equilibrium position,

Deflecting torque = Restoring torque,

$$N\,B\,I\,A = C\,\theta$$

$$\therefore\quad I = \left(\frac{C}{nAB}\right)\theta$$

But, C, n, A, B are constant

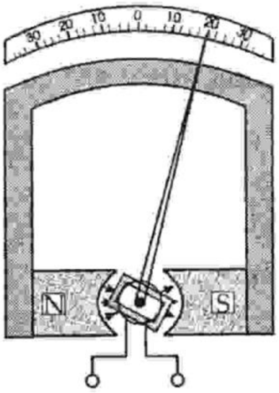
$$\therefore\quad I \propto \theta$$

Construction of MCG (Pivoted Type)

In pivoted type M.C.G. a coil is placed between the two poles of a horseshoe magnet having concave poles. The coil is mounted or a pivoted between the two supports. The supports are having jeweled bearings, which are almost frictionless.

2) Two hair springs one above and one below, the coil control the rotation of the coil. The two springs are spiraled in opposite direction. The current enters through one spring and leaves through the other.

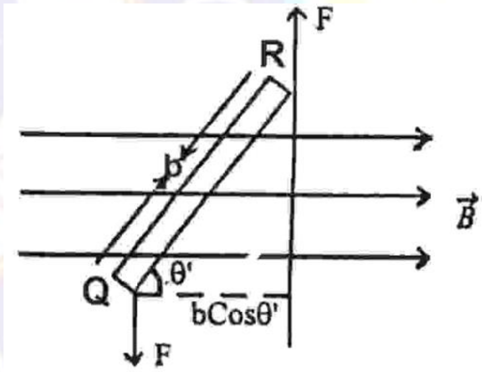
3) A long pointer is attached to the coil. It shows the rotation of coil on a scale.



4) The whole apparatus is fitted in a box having a window through which the deflection can be observed.

Why should the magnetic field in moving coil galvanometer be radial?

The radial magnetic field ensures that in any position, the plane of the coil is parallel to the field and current flowing through the coil is directly proportional to the deflection produced in the coil. i.e. we can have linear scale.



If this is not done, means, if the magnetic field is not radial, then plane of the coil makes certain angle with the direction of magnetic induction. As a result, a varying torque acts on the coil and the scale will not be linear. Let, in such a case, the deflecting torque is τ which is given by –

τ = n B I A cos θ'

Where, θ' = angle made by the plane of the coil with the direction of magnetic induction.

If C is the restoring torque per unit twist, then –

Restoring torque (τ') = Cθ

In equilibrium, τ = τ'

$\therefore$ n B I A cos θ' = Cθ

$$\therefore \quad I = \left(\frac{C}{nAB}\right)\frac{\theta}{\cos\theta'}$$

In such a case, no linear relation is between I and θ. To have linear relation between them, it is necessary that,

Cos θ' = 1

i.e θ' = 0^0.

Means, plane of the coil must be parallel to the direction of magnetic induction. In any position of the coil it is possible only when the magnetic field is radial.

Sensitivity of M. C. G.:

Sensitivity of M.C.G. is defined as the ratio of change in deflection to the change in current.

Therefore, Sensitivity

$$\text{Si} = \frac{d\theta}{dI}$$

A galvanometer is said to be sensitive, if it produces large deflection for a small current.

For a M. C.G.,

$$I = \left(\frac{C}{nAB}\right)\theta \text{ -------------- (1)}$$

Where, C $\rightarrow$ torque per unit twist.

n $\rightarrow$ no. of turns of coil.

A $\rightarrow$ Area of each turn.

B $\rightarrow$ Magnetic induction.

Diff. eqn (1) partially,

$$\therefore \quad \frac{dI}{d\theta} = \frac{C}{nAB}$$

$$\therefore \quad \frac{d\theta}{dI} = \frac{nAB}{C}$$

$$\therefore \quad \text{Si} = \frac{d\theta}{dI} = \frac{nAB}{C}$$

Thus, sensitivity of a M. C. G. can be increased by –

1) increasing no. of turns of coil (n)
2) increasing area of each turn of coil (A)
3) by increasing magnetic induction of the magnetic field (B)
4) decreasing restoring torque per unit twist of suspension fiber (C).

Accuracy of M.C.G.:

For a M. C. G.,

$$I = \left(\frac{C}{nAB}\right)\theta \text{ ------------ (1)}$$

Where, C $\rightarrow$ torque per unit twist.

n $\rightarrow$ no. of turns of coil.

A $\rightarrow$ Area of each turn.

B $\rightarrow$ Magnetic induction.

Diff. eqn (1) partially,

$$\therefore \quad dI = \left(\frac{C}{nAB}\right)d\theta \text{ -------- (2)}$$

By (2) ÷ (1)

$$\frac{dI}{I} = \frac{d\theta}{\theta}$$

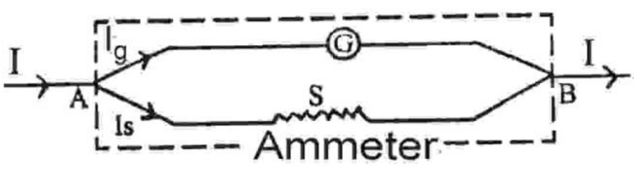
Where, $\frac{dI}{I}$ is representing error produced in the measurement of current. For maximum accuracy of M.C.G., error should be minimum.

Thus, for error to be minimum, deflection should be maximum. Hence, the accuracy of M.G.C. is maximum for large deflection.

Ammeter:

Ammeter is an electrical instrument used to measure current in an electric circuit. Introduction of ammeter in the circuit shouldn't change the original value of current. But, since it is connected in series in the circuit, high resistance of the coil of M. C. G. changes the current in the circuit if used directly for measuring current. Hence, M.C.G. can not measure current accurately, since current through circuit decreases.

Therefore, to measure the current



accurately, a low value resistance called shunt is connected in parallel with M.C.G. coil. It reduces resistance of M.C.G. Hence, the current can be measured accurately.

From above fig.

$I = I_g + I_s.$

$\therefore$ $I_s = I - I_g$ ---------- (1)

Where, I = current to be measured.

I_g = Current through galvanometer

I_s = current through shunt.

Let,

G = Resistance of galvanometer

S = shunt.

Since, G,S are connected in parallel,

$\therefore$ P. D. across 'G' = P. D. across S

$\therefore$ $V_G = V_S$

$\therefore$ $I_gG = I_sS$

Using eqn (1),

$I_gG = (I - I_g) S$

$$\therefore \quad S = \frac{I_g G}{I - I_g}$$

$$\therefore \quad S = \left(\frac{I_g}{I - I_g}\right) G \text{ --------- (2)}$$

Using Eqn (2), desired value of shunt can be calculated. By connecting which in parallel with galvanometer coil, it can be converted into ammeter to measure the current of value up to I.

Also, $I_s / I_g = G/S$

$$\therefore \quad \frac{I_s}{I_g} + 1 = \frac{G}{S} + 1$$

$$\therefore \quad \frac{I_s + I_g}{I_g} = \frac{G+S}{S}$$

$$\therefore \frac{I}{I_g} = \frac{G+S}{S}$$

$$\therefore \quad I_g = \left(\frac{S}{G+S}\right) I \text{ ---------- (3)}$$

Using equation (3), current flowing through galvanometer can be determined.

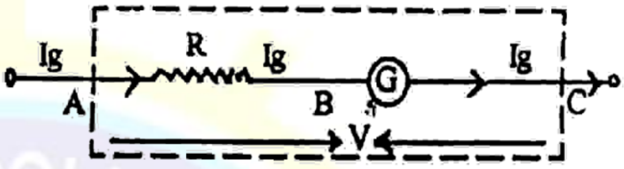
Voltmeter:

Voltmeter is an electrical instrument which is used to measure potential difference between two points in an electrical circuit. It is always connected in parallel in the circuit.

Introduction of Voltmeter shouldn't change the value of original P.D.

If a M. C. G. is introduced directly for the measurement of P. D., then due to division of current in the circuit, which flows through M.C.G., original value of P.D. changes.

For this, M.C.G. has to be converted into voltmeter by connecting a high resistance in series with galvanometer coil and P.D. can be measured accurately.



Let,

G = Resistance of galvanometer.

I_g = Current through galvanometer

R = High value series resistance.

V = P. D. to be measured.

From fig.

P.D. across AC is given by –

$V = V_{AB} + V_{BC}$

$\therefore$ $V = V_R + V_g$

$\therefore$ $V = I_gR + I_gG.$

$\therefore$ $V = I_g (R + G)$

$$\therefore \quad \frac{V}{I_g} = R + G$$

$$\therefore \quad R = \frac{V}{I_g} - G$$

By using above expression, the required value of high series resistance can be calculated to convert M.C.G. into voltmeter, to measure the P.D. up to V.

Explain why an ammeter must have a low resistance.

1) An ammeter is used for the measurement of current. For this purpose, the ammeter is connected in series with the circuit.

2) If the ammeter has a high resistance, the total resistance of the circuit increases when the ammeter is connected. The increase in resistance causes a lower current to pass through the circuit and this lower current is measured by the ammeter.

3) On the other hand, if the ammeter has a low resistance, the resistance of the circuit does not change appreciably when the ammeter is connected. Therefore almost the same current passes through the circuit and it is measured by the ammeter.
4) Thus for the accurate measurement of current, the ammeter must have a very low resistance.

Explain why a Voltmeter must have a very high resistance.
1) A voltmeter is used to measure the P.D. across any part of the circuit, across which the P.D. is to be measured.
2) If the voltmeter has a low resistance, a larger current passes through it. As a result of this, the current passing through the concerned part of the circuit decreases and hence the P.D. across it is lowered. This lower P.D. is measured by the voltmeter.
3) On the other hand, if the voltmeter has a very high resistance, a negligible current passes through it when it is connected. Therefore the current in the concerned part of the circuit remains almost unchanged and hence the P.D. across it remains almost the same as before and it is measured by the voltmeter.
4) Thus for the accurate measurement of P.D., the voltmeter must have a very high resistance.

Distinguish between moving coil ammeter and moving coil voltmeter.

Moving coil ammeter	**Moving coil Voltmeter**
1. It consists of a moving coil galvanometer with a shunt (i.e. low resistance) connected in parallel with its coil.	1. It consists of a moving coil galvanometer with a high resistance connected in series with its coil.
2. Its resistance is very low.	2. Its resistance is very high.
3. It is connected in series in the circuit.	3. It is connected in parallel across which P.D is to be found.
4. The range of measurement of current can be changed by changing the shunt resistance.	4. The range of measurement of P.D. can be changed by changing the series resistance.
5. To measure a larger current, a shunt of smaller resistance is required.	5. To measure a larger P.D. a larger series resistance is required.

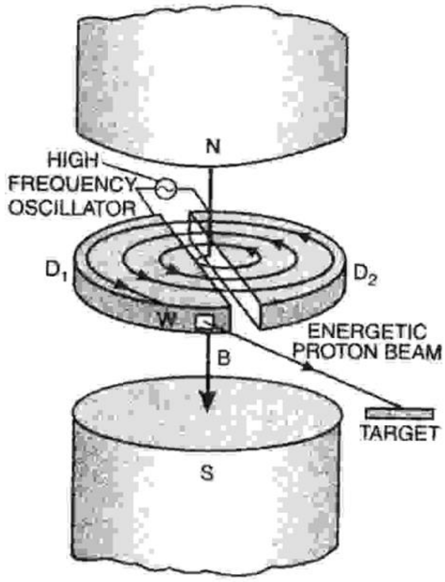
CYCLOTRON

Accelerator is used for accelerating charged particles, so that they acquire energy large enough to carry out the nuclear reactions.

Cyclotron was designed by Lawrence and Livingstone in 1931 in order to overcome the drawbacks of the linear accelerator. It is suitable for accelerating heavy charged particles such as protons, α-particles and positive ions. In a cyclotron, the positive ions cross again and again the same alternating (radio frequency) electric field and thereby again the energy. It is achieved by making them to move along spiral circular paths under the action of a strong magnetic field. It is also known as *magnetic resonance accelerator*.

Principle: It is based on the principle that *a positive ion can acquire sufficiently large energy with a comparatively smaller alternating potential difference by making it to cross the same electric field time and again by making use of a strong magnetic field.*

Construction:



It consists of two D-shaped hollow semicircular metal chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} mm of Hg. The whole apparatus is placed between the two poles of a strong electromagnet NS as shown in fig. The magnetic field acts perpendicular to the plane of the dees.

The positive ions are produced in the gap between the two dees by the ionization, of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas used.

Theory: Consider that a positive ion is produced at the centre of the gap at the time, when the dee D_1 is at positive potential and the dee D_2 is at a negative potential. The positive ion will move from dee D_1 to dee D_2. As the magnetic field acts normally to the motion of the positive ion, the position ion experiences force. The force on the positive ion due to magnetic field provides the centripetal force to the positive ion and it is deflected along a circular path. If ,

B = strength of the magnetic field
m = mass
v= velocity
and q =charge of the positive ion

then,

$$Bq\upsilon = \frac{m\upsilon^2}{r} \text{ ------------ (1)}$$

Here, r is the radius of the semi-circular path along which the positive ion will move inside the dee D_2.

$$\text{Thus, } \quad r = \frac{m\upsilon}{Bq} \text{ ----------------- (2)}$$

After moving along the semi-circular path inside the dee D_2, the positive ion reaches the gap between the two dees. At this stage, the polarity of the dees just reverses due to alternating electric field i.e. dee D_1 becomes negative and dee D_2 becomes positive. The positive ion again gains energy, as it is attracted by the dee D_1. After moving along the semi-circular path inside the dee D_1, the positive, ion again reaches the gap and it again gains the energy. This process repeats itself again and again. It is because, *the positive ion spends the same time inside a dee irrespective of its velocity or the radius of the circular path.*

The time spent inside a dee to cover semi-circular path,

$$t = \frac{\textbf{lenght of the semi circular path}}{\textbf{velocity}} = \frac{\pi r}{\upsilon}$$

Using equation (2), we have

$$t = \frac{\pi m}{Bq}$$

As required, the time which a positive ion of mass m and charge q spends inside a dee is indeed independent of its velocity and radius of the semi-circular path. It may again be pointed out that decrease in time spent inside a dee due to incrasing velocity of positive ions is exactly compensated by the increase in length of the semi-circular path. Due to this condition, the positive ion always crosses the alternating electric field across the gap between the two dees in correct phase.

NUMERICALS

1. Combined resistance of galvanometer of 495Ω& its shunt, is 9.9Ω. Find shunt resistance.
2. Combined resistance of galvanometer of 500 Ω & its shunt is 25Ω. Calculate value of shunt resistance.
3. A galvanometer having resistance of 49.9Ω gives full scale deflection with current of 10 mA How it can be converted into Ammeter to measure a current of 5 A.
4. Millimeter of resistance 10Ω has rage of 0.25mA . How would be convert it into voltmeter reading up to 25 volt.
5. Certain voltmeter has a range of 15 volt & resistance 1000Ω. How will you use it to measure a voltage up to 150 volt.
6. Galvanometer carries a maximum current of 15mA. When voltage of 75 mV is applied. Convert this into voltmeter to read upto 150 V & into ammeter to read upto 25 A.

7. Galvanometer has current sensitivity of 10 div. per mA & voltage sensitivity 1division per mV. The instrument has 50 divisions. How will you use it as an ammeter to read maximum of 5A & as voltmeter to read maximum of 50 V.
8. Galvanometer has current sensitivity of X div. per mA & voltage sensitivity of X/2 div per V. If instrument has y divisions. Find resistance of Galvanometer.
9. A rectangular Coil of effective area 0.1 m^2 if suspended freely in radial magnetic field of
   0.01 wb/m^2. If the torque per unit twist of suspension fibre is 5×10^{-9} Nm/degree. Find angle through which coil rotates when current of 200 mA in passed through it.
10. Rectangular coil of moving coil galvanometer has 50 turns & area of 12 cm^2 suspended in radial magnetic field of induction 0.025 wb/m^2. Torsional constant of suspension fibre is 1.5×10^{-7} Nm/degree. Calculate current required to produce deflection of 0.5^0.
11. A rectangular coil of 100 turns each of area $10cm^2$ hang Freely in radial magnetic field. Coil deflects through angle of 30^0 when current of 0.5 mA is passed through it. If torsional constant is 25×10^{-9} Nm/ rad. Find magnetic field.
12. Galvanometer is shunted by $\left(\frac{1}{r}\right)^{th}$ of its resistance. Find fraction of total current passing through the galvanometer.
13. Resistance of galvanometer is 4052 Ω. What shunt must be connected so that 1% of total current will flow through galvanometer.
14. Coil of M.C.G. has 100 turns & Effective Area 0.05 m^2. If suspended in radial magnetic field of induction 0.01 wb/m^2. Torque per unit twist
    5×10^{-9} Nm/degree. Find sensitivity of M.C.G.
15. Current of 0.1 Amp produces deflection 60^0 in T.G. What current will produce deflection of 30^0. How much deflection must be produced by current of 173.2 mA.
16. A T .G has 50 turns each of radius 6.28 cm. If $B_H = 3.6 \times 10^{-5}$ wb/m^2. Find reduction factor of T.G. Find the deflection due to 12 mA current.
    Given: $\mu_0 = 4\pi \times 10^{-7}$ S.I. unit
17. Two T.G. are identical in all respect. One has two turns & other has twenty turns. If connected in parallel across source of emf. Find relation between deflections observed by them.
18. A.T.G. of 50 turns and radius 10 cm connected in series with another T.G. of 100 turns of radius 15 cm. Same current flow through them. if deflection in 1st T.G. is 45^0. Calculate deflection in other.
19. A coil of T.G. having diameter of 16 cm is set up in Magnetic Meridian and current of 0.5 A is passed through the coil. If deflection of needle is 60^0. Find length of wire used in the winding of coil. $B_H = 2 \times 10^{-5}$ wb/m^2.
20. Find current sensitivity of T.G. when deflection is 60^0. No. of turns = 50. Mean Radius = 10 cm. $B_H = 5\times 10^{-5}$ wb/m^2.
21. Voltmeter has 1 kΩ resistance, sensitivity 1V per div. Find res. inserting so as to read
    5V per div.

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