

Circular motion is motion of a body or a particle along circumference of a circle with fixed centre and constant radius.

TYPES OF CIRCULAR MOTION:

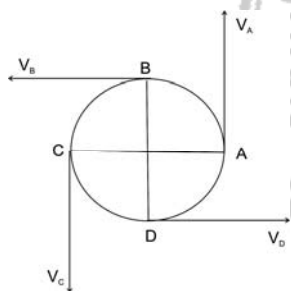
Circular motion can be classified into two types-

- I) Uniform circular motion (U.C.M)
- II) Non Uniform circular motion (Non - U.C.M)

I) Uniform circular motion (U.C.M)

- i) Stone whirled in horizontal plane at the end of the string.
- ii) Hands of a clock
- iii) Blades of a fan

In each of these cases, every particle moves in a circular motion. As the particle moves in a circle it not only sweep out angle θ but it also cover arc length 's'. The tangential velocity indicates how fast the particle



actually moves. The angular velocity of the particle indicates how fast the particle changes its angular position. Uniform circular motion is a special case of

circular motion where particle or body will cover equal distance AB, BC, CD & DA in equal interval of time i.e. speed is constant during its motion. If the magnitude of instantaneous linear velocity which is tangential at that point remains constant at any position then such a motion is called U.C.M.

$$v_A = v_B = v_C = v_D$$

$$\text{But, } \vec{v}_A \neq \vec{v}_B \neq \vec{v}_C \neq \vec{v}_D$$

DEFINITION OF U.C.M.: It is the motion of the body or a particle along the circumference of the circle with constant speed(magnitude of velocity).

In such a motion direction of linear velocity changes continuously but its magnitude remains constant.

II) Non Uniform circular motion (Non - U.C.M)

DEFINITION OF NON U.C.M.:

It is the motion of the body or a particle along the circumference of the circle with variable velocity.

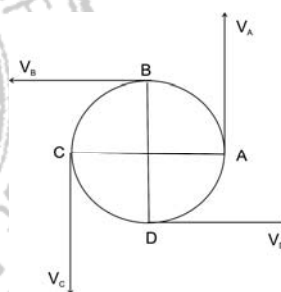
In case of a non U.C.M .magnitude as well as direction of linear velocity of the object changes. Therefore body possess

acceleration i.e. a_T as well as centripetal

acceleration i.e. a_c which are at right angles to each other. The total acceleration of the body in the case given by,

$$a = \sqrt{a_T^2 + a_c^2}$$

Note: In a U.C.M. tangential acceleration is zero.



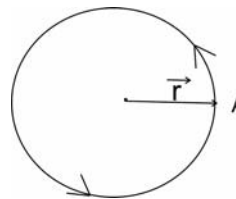
$$v_A \neq v_B \neq v_C \neq v_D$$

as well as

$$\vec{v}_A \neq \vec{v}_B \neq \vec{v}_C \neq \vec{v}_D$$

Motions of particle in vertical circle, Starting or retarding motion of a fan etc. Are the examples of non U.C.M.

RADIUS VECTOR:



The position of the particle performing U.C.M. can be represented at any instant by a

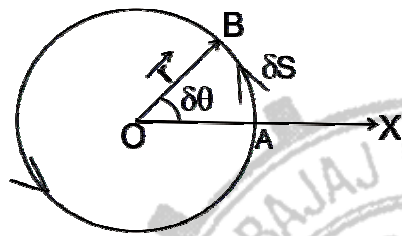
vector drawn from the centre of the circle to the position of the particle. This is called radius vector or position vector of the particle at a given time.

In figure, OA is the radius vector when the particle is at A.

CHARACTERISTICS:

- 1) As the particle revolve, radius vector rotates continuously along it.
- 2) It is directed away from the center of the circle.
- 3) It is always perpendicular to velocity vector.
- 4) Direction of radius vector changes continuously but its magnitude remains constant, equal to radius of circle.

ANGULAR DISPLACEMENT:



Consider a particle moving in an anticlockwise sense along the circumference of the circle of radius r , OX is a fixed reference line in the plane of the circle. Let A be the initial position of the particle, when it was on the reference line. 'B' is its position at time δt . The angle which the radius vector of the particle makes at an instant with the reference line OX is called angular displacement of particle at that instant w.r.t. fixed reference line OX

In figure angular displacement of the particle at time δt
 $= \angle AOB$

Magnitude of angular displacement

$$\delta \theta = \text{angle in radian}$$

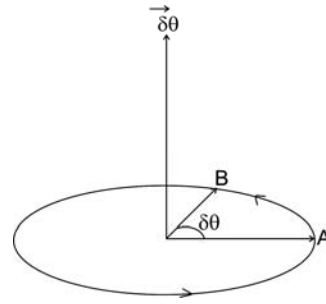
$$\delta \theta = \frac{\text{arc AB}}{\text{radius OA}}, \delta \theta = \frac{\delta s}{r}$$

S.I. unit = radian and dimensionless quantity i.e. $[M^0 L^0 T^0]$

Infinitely small angular displacement is vector quantity and its direction given by right hand thumb rule i.e.

Curved fingers are in the direction of revolution of particle then the thumb

points in the direction of angular displacement along the axis of revolution.



Finite angular displacement is **not vector quantity** as they do not obey the commutative and associative law of vector addition.

ANGULAR VELOCITY (ω):

It is the rate of change of angular displacement with time.

If θ and $\theta + \delta \theta$ are the angular displacement at time 't' and $t + \delta t$ respectively then the average angular velocity ' ω ' in the time interval δt is given by.

$$\omega = \frac{\delta \theta}{\delta t}$$

In the limit, $\delta t \rightarrow 0$

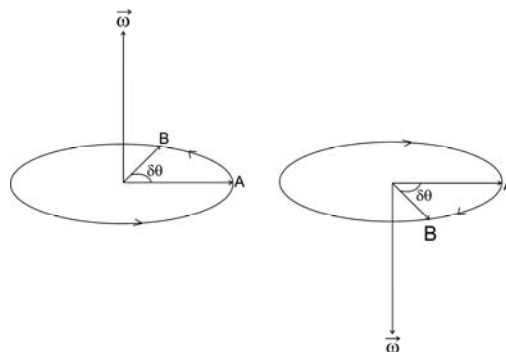
$$\omega = \lim_{\delta t \rightarrow 0} \frac{\delta \theta}{\delta t}$$

$$\omega = \frac{d\theta}{dt}$$

S.I. unit \rightarrow radian/sec

Dimension $\rightarrow [M^0 L^0 T^{-1}]$

Its direction is given by right hand thumb rule as shown in the figure.



ANGULAR ACCELERATION (α):

It is the rate of change of angular velocity with time.

If ' ω ' and ' $\omega + \delta \omega$ ' are the angular velocities in time ' t ' and ' $t + \delta t$ ' respectively then the average angular acceleration ' α ' in the time interval ' δt ' is given by

$$\alpha = \frac{\delta \omega}{\delta t}$$

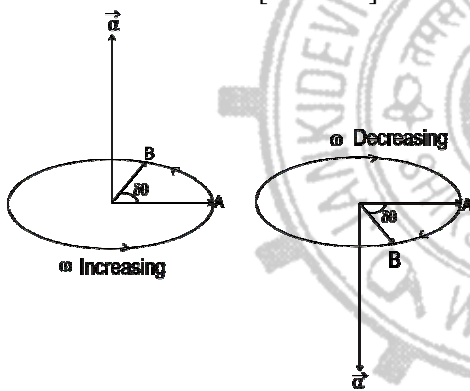
In the time $\lim \delta t \rightarrow 0$

$$\alpha = \lim_{\delta t \rightarrow 0} \frac{\delta \omega}{\delta t}$$

$$\alpha = \frac{d\omega}{dt}$$

S.I. unit \rightarrow radian/sec²

Dimension $\rightarrow [M^0 L^0 T^{-2}]$

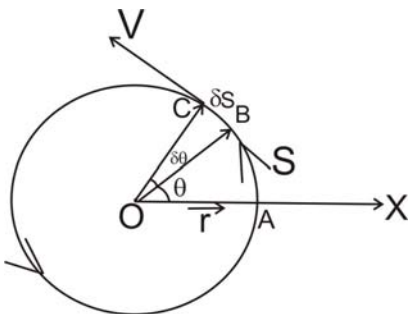


For a U.C.M. since ' ω ' is constant.

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha = 0$$

RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY:



Consider a particle performing U.C.M. in an anticlockwise sense along the circumference of the circle of radius ' r '. Let ' v ' and ' ω ' denote linear and angular velocities respectively. At time ' t ', angular displacement of the particle w.r.t. reference line OX when the particle moves from A to B is given by,

$$\theta = \frac{s}{r} \quad \text{or} \quad s = r \cdot \theta$$

Let $\delta \theta$, δs be small increments of quantities, in θ and s respectively in the time interval δt .

$$\text{Then, } \delta s = r \cdot \delta \theta$$

Dividing both sides by δt

$$\frac{\delta s}{\delta t} = r \cdot \frac{\delta \theta}{\delta t}$$

The limit when $\delta t \rightarrow 0$

$$\lim_{\delta t \rightarrow 0} \frac{\delta s}{\delta t} = r \cdot \lim_{\delta t \rightarrow 0} \frac{\delta \theta}{\delta t}$$

$$\frac{ds}{dt} = r \cdot \frac{d\theta}{dt}$$

$$V = r \cdot \omega \quad (\text{scalar relation})$$

Vector relation

$$\vec{v} = \vec{\omega} \times \vec{r} \quad (\text{vector relation})$$

PERIOD TIME OR TIME PERIOD (T):

It is the time taken by the particle to complete one revolution or one revolution.

S.I. unit \rightarrow seconds.

FREQUENCY OF ROTATION (f OR n):

It is the number of revolution taken by the particle per second.

S.I. unit \rightarrow r.p.s. or Hertz (Hz)

RELATION BETWEEN TIME PERIOD AND ANGULAR VELOCITY

Time period is the time taken by the particle to complete one rotation.

$$\text{Linear velocity} = \frac{\text{linear distance travelled}}{\text{time taken}}$$

For 1 rotation,

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v}$$

Substituting $v = r\omega$

$$T = \frac{2\pi}{r\omega}$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{1}{f}$$

$$\frac{1}{f} = \frac{2\pi}{\omega}$$

$$\omega = 2\pi f$$

RELATION BETWEEN ANGULAR ACCELERATION AND TANGENTIAL ACCELERATION OR LINEAR ACCELERATION:

$$a_T = \frac{d(r\omega)}{dt}$$

$$a_T = r \frac{d\omega}{dt}$$

$$a_T = r\alpha$$

Equation for uniformly accelerated rotational motion:

Linear Motion	Angular Motion
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

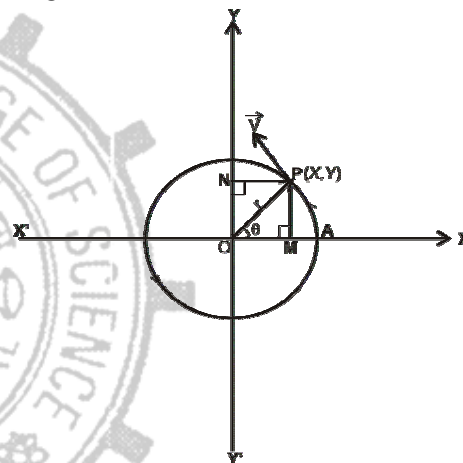
Where ω_0 , is the angular velocity at time $t = 0$, θ Represent angular displacement and ω angular velocity at time 't'.

Q.OBTAIN AN EXPRESSION FOR CENTRIPETAL OR RADIAL ACCELERATION

When a particle performs U.C.M., its linear velocity at any instant is along the tangent to the circle. Its magnitude remains constant but its direction is changing with time. Hence, due to this change in velocity, the particle acquires an acceleration which is along the radius of the circle directed towards centre. Therefore this acceleration is called **radial or centripetal acceleration**.

CALCULUS METHOD:

We select co-ordinate axis in such a way that the circle lies in the X – Y plane and the origin 'O' coincides with the centre of



the circle. Suppose at time $t = 0$, the particle is at 'A' along the X – Axis. At time 't' at P. Then the angular displacement

of the particle is $\theta = \omega t$

Draw $PM \perp OX$ and $PN \perp OY$

From figure,

$$x = r \cos \theta = r \cos (\omega t)$$

$$y = r \sin \theta = r \sin (\omega t)$$

The position vector of the particle is given By-

$$\vec{r} = X\vec{i} + Y\vec{j}$$

$$\vec{r} = r \cos \omega t \vec{i} + r \sin \omega t \vec{j} \text{ ----- (i)}$$

(\vec{i} and \vec{j} are unit vector along x and y axis)

Differentiating \vec{r} w.r.t. time, t

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d}{dt} (r \cos \omega t \vec{i} + r \sin \omega t \vec{j})$$

$$\vec{v} = -r\omega \sin \omega t \vec{i} + r\omega \cos \omega t \vec{j}$$

Differentiating w.r.t time 't'

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = -r\omega^2 \cos \omega t \vec{i} - r\omega^2 \sin \omega t \vec{j}$$

$$\vec{a} = -\omega^2 (r \cos \omega t \vec{i} + r \sin \omega t \vec{j})$$

Using (i)

$$\vec{a} = -\vec{r} \omega^2$$

Negative sign indicates that \vec{a} and \vec{r} have opposite direction i.e. acceleration is directed towards the center of the circle. Hence, acceleration is centripetal acceleration. Magnitude of centripetal acceleration is given by-

$$a = \sqrt{a_x^2 + a_y^2}$$

$$a = \sqrt{r^2 \omega^4 \cos^2 \omega t + r^2 \omega^4 \sin^2 \omega t}$$

$$a = \sqrt{r^2 \omega^4 (\cos^2 \omega t + \sin^2 \omega t)}$$

$$a = \sqrt{r^2 \omega^4}$$

$$a = r \omega^2 \quad \text{----- (1)}$$

$$a = (r\omega) \omega$$

Substituting $r\omega = V$

$$a = V\omega \quad \text{----- (2)}$$

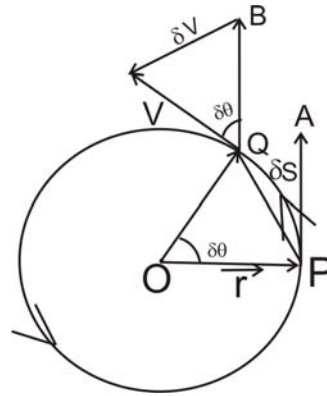
Substituting, $\omega = \frac{V}{r}$ in equation (1)

$$a = r \cdot \frac{V^2}{r^2}$$

$$a = \frac{V^2}{r} \quad \text{----- (3)}$$

Equation (1), (2) and (3) are expression for centripetal acceleration in different forms

ALTERNATIVE METHOD:



Consider a particle performing U.C.M. in an anticlockwise sense along the circumference of the circle of radius 'r'. Let 'v' and ' ω ' represent linear and angular velocity respectively. Let the particle move from P to Q in a very short time interval ' δt '. Let ' $\delta\theta$ ' be the angular displacement in time ' δt '; velocities of the particle at P and Q are shown respectively by \vec{PA} and \vec{QC} . Draw

\vec{QB} parallel and equal to \vec{PA} and draw \vec{BC} in the direction \vec{BC} .

By triangle law of vector addition.

$$\vec{QB} + \vec{BC} = \vec{QC}$$

$$\vec{BC} = \vec{QC} - \vec{QB} = \vec{QC} - \vec{PA}$$

\vec{BC} = velocity at Q - velocity of particle at P.

\vec{BC} = Change in velocity (δv) in time interval δt .

When the radius vector rotates through angle $\delta\theta$, the tangent to the radius vector also rotates through the same angle $\delta\theta$.

Hence in the isosceles triangle OPQ and QBC are similar triangles,

$$\frac{BC}{PQ} = \frac{QB}{OP}$$

$$\therefore \frac{BC}{QB} = \frac{PQ}{OP}$$

$$\frac{\delta v}{v} = \frac{PQ}{r}$$

For small time 'δ t' arc length δ s is equal to distance PQ

$$\frac{\delta v}{v} = \frac{\delta s}{r}$$

$$\delta v = \frac{v \cdot \delta s}{r}$$

Dividing by δ t.

$$\frac{\delta v}{\delta t} = \frac{v}{r} \cdot \frac{\delta s}{\delta t}$$

In the limit, when δ t → 0

$$\lim_{\delta t \rightarrow 0} \frac{\delta v}{\delta t} = \frac{v}{r} \quad \lim_{\delta t \rightarrow 0} \frac{\delta s}{\delta t}$$

$$\frac{dv}{dt} = \frac{v}{r} \cdot \frac{ds}{dt}$$

$$a = \frac{v^2}{r} \text{ ----- (1)}$$

Substituting v = r ω in equation (1)

$$a = \omega^2 r \text{ ----- (2)}$$

Substituting $\frac{v}{r} = \omega$ in equation (1)

$$a = v \omega \text{ ----- (3)}$$

Equation (1), (2), (3) are the expressions of centripetal acceleration in different forms. The direction of 'a' is same as that of δ v.

As δ t → 0, δ θ → 0, P approaches Q δ v becomes perpendicular to the tangent i.e. acts along the radius towards the centre of the circle.

Characteristics of UCM:

- 1) Body or particle repeatedly passes through any point in the path of its motion in a fixed interval of time.
- 2) Angular velocity of the particle and kinetic energy of the particle remains constant.
- 3) Circular motion requires centripetal acceleration acting on the body or the particle.
- 4) Direction of position vector or radius vector changes continuously but its magnitude always remains constant equal to radius of the circle.
- 5) Direction of velocity which acts along the tangent to the path changes continuously but its magnitude remains constant.
- 6) Direction of acceleration vector which acts along the radius of the circle towards the centre changes continuously but its magnitude is constant.

CENTRIPETAL FORCE:

When a stone is tied to a string and rotated in a horizontal circle with constant speed 'v' the magnitude of velocity remains constant but its direction changes continuously. A force is necessary to change its direction. If there were no force the stone would move straight. Therefore, external force must act to keep the stone in circular path. This constant inward force must not change the magnitude of velocity therefore it must act at right angles to the direction of motion i.e. along the radius of the circle. This inward force is called centripetal force.

DEFINITION: Centripetal force is the force which is radially directed towards the centre of the circle and which keeps the particle moving in constant speed along a circular path.

The magnitude of Centripetal force $F_c = \text{mass} \cdot \text{acceleration}$

$$= m \cdot \frac{v^2}{r}$$

$$= m r \omega^2$$

$$= m v \omega$$

In vector form, it can be given as-

$$\vec{F}_c = - \frac{m v^2 \cdot \vec{r}_0}{r}$$

When \vec{r}_0 a unit vector along the radius vector and negative sign indicates that \vec{F}_c and \vec{r}_0 are oppositely directed.

Note: Centripetal force and instantaneous displacement being perpendicular, **no work is done by centripetal force.**

$$\begin{aligned} dw &= \vec{F} \cdot d\vec{s} \\ &= F ds \cos \theta \\ &= F ds \cos 90^\circ \quad (\theta = 90^\circ) \\ &= 0 \\ dw &= 0 \end{aligned}$$

Centripetal force is a real force as it is due to known interactions and is always provided by an external source.

e.g: 1) When a satellite is orbiting round the earth, the centripetal force required for its circular motion is provided by gravitational force of attraction.

$$\frac{mv^2}{r} = \frac{G Mm}{r^2}$$

e.g: 2) when a car moves around a horizontal circular road, the centripetal force required for the circular motion of the car is provided by the frictional force between the road surface and the tyres.

$$\frac{mv^2}{r} = F_s$$

e.g. 3) when electron revolves around the nucleus in circular orbit, necessary centripetal force is provided by electrostatic force of attraction between positively charged nucleus and negatively charged electrons.

$$\frac{mV^2}{r} = \frac{1}{4\pi\epsilon_0 k} \cdot \frac{e^2}{r^2}$$

CENTRIFUGAL FORCE:

A body is placed on a rotating turn table with one end of a string tied to the body and the other end is fixed at the centre of the turn table. Observer 'A' who is in inertial frame of reference finds that the body moves in a circle and the centripetal force necessary is provided by the tension in the string. Observer 'B' is on the rotating turn table and hence in non inertial frame of reference. He finds that the body is at rest. This is because the body is lying at the same distance all the time as the body remains at rest. He concludes that the body is acted upon by a radially outward force which is equal and opposite of the tension in the string. This pseudo force which is equal and opposite to the centripetal force is called **centrifugal force**.

DEFINITION: Centrifugal force is a radially outward pseudo force which is exerted on a body performing U.C.M. as observed by the observer in the non – inertial frame of reference.

Magnitude is same as that of centripetal force i.e. mv^2/r but its direction is radially away from the centre of the circle.

Example:

- 1) When a bus is taking a turn on a horizontal curved road, the passengers experience a centrifugal force which pushes them away from the centre of the curve.
- 2) A cyclist while taking a turn on a horizontal curved road inclines himself to the inside of the circle because he experiences a centrifugal force.

APPLICATIONS OF CENTRIFUGAL FORCE:

1. In centrifuges due to the centrifugal action, the suspended particles in the liquid get separated when the vessel containing the liquid spins fast about an axis.
2. This principle is used in separating cream from milk, drying wet clothes, etc.
3. Centrifugal pump used to lift water, works on the principle of centrifugal force.

Q. Distinguish between Centripetal Force & Centrifugal Force

Centripetal Force	Centrifugal Force
1) It is a real force.	1) It is a pseudo force.
2) It is directed radially towards the centre of the circle.	2) It is directed radially away from the centre of the circle
3) It has a source which arises due to known interactions of nature gravitational, nuclear or electromagnetic.	3) It has no obvious source and arises due to the acceleration of the frame of reference.
4) They arise in both inertial and non inertial frame of reference.	4) They arise only in non inertial frame of reference.
5) It is necessary to maintain U.C.M.	5) It is not required for U.C.M.

Q. Why banking of roads is necessary?

When a car moves on a curved horizontal road friction between the road surface and the tyre provide the necessary centripetal force.

$$\frac{mv^2}{r} = F_s$$
$$F_s = \mu N$$

Where,

μ = coefficient of friction and N = Normal reaction

$$\therefore \frac{mv^2}{r} = \mu N$$
$$= \mu mg$$

$$(\because N = mg)$$

$$v^2 = \mu rg$$

$$v_{\max} = \sqrt{\mu rg}$$

This is the maximum speed with which a car can be safely driven along a curved horizontal road.

However friction is an unreliable factor, because at high speed or when the road surface is wet or when oil is spilled over the road surface, the frictional force is insufficient to provide the necessary centripetal force. Lack of adequate frictional force may therefore result in skidding or overturning of cars.

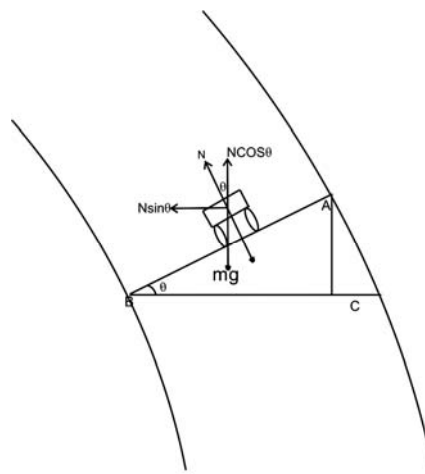
To avoid this, the force of friction may be increased by making the road surface rough. This may result in wear and tear of tyre.

To avoid dependence on friction and to provide the necessary centripetal force the outer edge is raised above the inner edge. This arrangement of keeping the road surface, inclined to the horizontal is called **banking of road**.

The angle between the inclined road surface and the horizontal is called the angle of banking.

Q. What is angle of banking? Obtain an expression for angle of banking.

To avoid dependence on friction and to provide the necessary centripetal force the outer edge is raised above the inner edge. This arrangement of keeping the road surface, inclined to the horizontal is called **banking of road**.



The angle between the road surface and the horizontal is called the angle of banking.

Figure represents a road surface AB banked at angle θ with the horizontal. Consider a car, of mass m , moving, with a uniform speed ' v ' along the curved road, of radius ' r ' forces acting on the car are:

- 1) Weight mg vertically downwards
- 2) Normal reaction N

N can be resolved into two rectangular components

- 1) Vertical component $N \cos \theta$
- 2) Horizontal component $N \sin \theta$

As the horizontal component $N \sin \theta$ is towards the centre, it provides the centripetal force.

$$N \sin \theta = \frac{m v^2}{r} \quad \text{---- (1)}$$

$$N \cos \theta \text{ balances weight } mg$$
$$N \cos \theta = mg \quad \text{---- (2)}$$

Dividing equation (1) by (2)

$$\tan \theta = \frac{v^2}{rg} \quad \text{---- (3)}$$

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

The above equation gives the angle of banking of the road surface. So that a car can move with a speed 'v' without dependence on friction. As there is no mass term, angle of banking is independent of the mass.

Factors which decide the angle of banking are-

- Maximum speed of the car.
- Radius of curvature of the road.
- Acceleration due to gravity.

Also from equation (3)

$$V^2 = rg \tan \theta$$

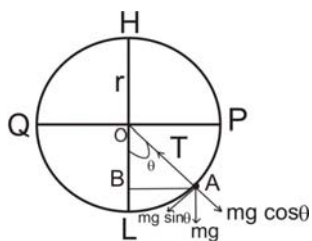
$$V_{\max} = \sqrt{rg \tan \theta}$$

This is the maximum speed with which a car can be safely driven without dependence on friction.

MOTION IN VERTICAL CIRCLE:

Motion in vertical is an example of non- U.C.M.

Consider a body of mass 'm' is tied to one end of a string and is whirled in a vertical circle of radius 'r' as shown in fig. The motion is circular but not uniform, since body moves from lowest point 'L' to highest point 'H' against the gravity and with change in direction its speed decreases, while moving from highest point 'H' to lowest point 'L' since body moves with gravity with change in direction its speed increases.



EXPRESSION FOR TENSION IN THE STRING:

Let, the body is at point 'A' at any instant 't' making an angle θ with lowest point 'L'. Let 'T' is the tension in the string. Weight 'mg' can be resolved into two components-

- $mg \cos \theta$
- $mg \sin \theta$

Component $mg \cos \theta$ is directed opposite to 'T' hence net force in the string towards the centre is given by –

$$F = T - mg \cos \theta$$

This force provides necessary centripetal force. Therefore,

$$\frac{mv^2}{r} = T - mg \cos \theta \text{ -----(1)}$$

$$T = \frac{mv^2}{r} + mg \cos \theta \text{ ----- (2)}$$

At highest point 'H':

At point 'H' $\theta = 180^\circ$

$$\therefore \cos \theta = -1$$

$$\frac{mv^2}{r} = T_H + mg \text{ -----(3)}$$

\therefore centripetal force = maximum

$$T_H = \frac{mv^2}{r} - mg \text{ ----- (4)}$$

At lowest point 'L':

At point 'L' $\theta = 0^\circ$

$$\therefore \cos \theta = 1$$

$$\frac{mv^2}{r} = T_L - mg \text{ ----- (5)}$$

\therefore centripetal force = minimum

$$T_L = \frac{mv^2}{r} + mg \text{ ----- (6)}$$

EXPRESSION FOR VELOCITY AND ENERGY AT DIFFERENT POINTS:

At highest point 'H':

From (3) tension at point 'H' is given by-

$$T_H = \frac{mv^2}{r} - mg$$

If $T_H > 0$ the string remains tight, body complete the circle, but when $T_H < 0$ or negative the string becomes loose and the body no longer remain on circular path. Therefore, for just completing vertical circle $T_H = 0$, then velocity at point 'H' is called critical velocity. Thus,

$$0 = \frac{mv_H^2}{r} - mg$$

$$\frac{mv_H^2}{r} = mg$$

$$\therefore v_H^2 = gr$$

$$v_H = \sqrt{gr} \text{ -----(7)}$$

This is the expression for velocity at highest point 'H'.

At lowest point 'L':

According to law of conservation of total energy at point 'H' and at point 'L' remains the same. Therefore,

$$\begin{aligned} \text{T.E at point 'L'} &= \text{T.E at point 'H' i.e.} \\ \text{K.E. at point 'L'} + \text{P.E at point 'L'} &= \text{K.E. at point 'H'} + \text{P.E at point 'H'} \end{aligned}$$

$$\begin{aligned} \text{But at point 'L' P.E} &= 0 \\ \therefore \text{K.E. at point 'L'} &= \text{K.E. at point 'H'} + \text{P.E at point 'H'} \end{aligned}$$

$$\therefore \frac{1}{2}mv_L^2 = \frac{1}{2}mv_H^2 + mgh$$

But, at point 'H' $h = 2r$

$$\therefore \frac{1}{2}mv_L^2 = \frac{1}{2}mv_H^2 + 2rmg$$

$$\therefore v_L^2 = v_H^2 + 4gr$$

$$\text{But, } v_H = \sqrt{gr}$$

$$\therefore v_L^2 = gr + 4gr$$

$$v_L^2 = 5gr$$

$$v_L = \sqrt{5gr} \text{ -----(8)}$$

This is the expression for velocity at point 'L'.

Energy at point 'H':

Total energy at point 'H',

$$E_H = \text{K.E} + \text{P.E}$$

$$E_H = \frac{1}{2}mv_H^2 + mgh$$

Substituting $h = 2r$, $v_H = \sqrt{gr}$

$$E_H = \frac{1}{2}mgr + 2mgr$$

$$E_H = \frac{5}{2}mgr \text{ -----(9)}$$

Energy at point 'L':

Total energy at point 'L'

$$E_L = \text{K.E} + \text{P.E}$$

But, at point 'L' P.E. = 0

$$\therefore E_L = \text{K.E}$$

$$E_L = \frac{1}{2}mv_L^2$$

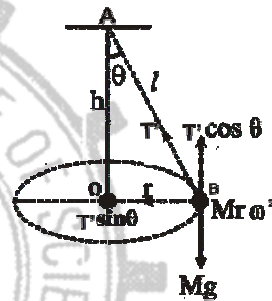
Substituting, $v_L = \sqrt{5gr}$

$$E_L = \frac{5}{2}mgr \text{ -----(10)}$$

Thus, expression for total energy at point 'H' & at point 'L' are same.

Conical Pendulum

When a metallic sphere called 'bob' is suspended from the rigid support with the help of thread or string and rotated in horizontal circle then it forms a cone. Such pendulum is called a 'Conical Pendulum'.



Let a bob of mass 'm' is suspended from the rigid support having its height 'h' and slant distance (l) it is rotated in horizontal circle then it forms a cone. At any position 'B' its semi vertical angle is ' θ ' the tension produced in the string at this point can be resolved into two components.

- i) $T' \cos \theta$ ii) $T' \sin \theta$

The vertical component $T' \cos \theta$ balances the weight of the bob. Therefore,

$$T' \cos \theta = mg \text{ -----(1)}$$

The horizontal component $T' \sin \theta$ is observed to be directed forwards the centre of horizontal circle hence it provides necessary centripetal force.

$$T' \sin \theta = m r \omega^2 \text{ -----(2)}$$

By (2) \div (1)

$$\frac{T' \sin \theta}{T' \cos \theta} = \frac{mr\omega^2}{mg}$$

$$\tan \theta = \frac{r\omega^2}{g}$$

$$\omega^2 = \frac{g \tan \theta}{r}$$

$$\omega = \sqrt{\frac{g \tan \theta}{r}} \text{ ----- (3)}$$

eqⁿ (3) gives the expression for angular velocity of the pendulum.

Let, T be the time period of pendulum

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

Using eqⁿ (3)

$$T = \frac{2\pi}{\sqrt{\frac{g \tan \theta}{r}}}$$

$$T = 2\pi \sqrt{\frac{r}{g \tan \theta}} \text{ ----- (4)}$$

In triangle OAB,

$$\frac{OB}{OA} = \tan \theta$$

$$\frac{r}{h} = \tan \theta$$

$$\therefore \frac{r}{\tan \theta} = h$$

Substituting in (4)

$$T = 2\pi \sqrt{\frac{h}{g}} \text{ ----- (5)}$$

Again in triangle OAB,

$$\frac{OA}{AB} = \cos \theta$$

$$\frac{h}{l} = \cos \theta$$

$$h = l \cos \theta$$

Substituting in (5)

$$T = 2\pi \sqrt{\frac{l \cos \theta}{g}} \text{ ----- (6)}$$

Equation (4), (5), (6) gives the expressions for time period of conical pendulum in different forms.

1. Object of mass 0.4 kg is whirled (rotated) in a horizontal circle of radius 2m. If it performs 60 rpm. Calculate centripetal force acting on it.
2. A 0.5 kg mass is rotated in horizontal circle of radius 20 cm. Calculate centripetal force acting if its angular speed of rotation is 0.6 rad/s.
3. Frequency of particle performing circular motion changes from 60 rpm to 180 rpm in 20 s. Calculate angular acceleration.
4. Determine angular acceleration of a rotating body which slows down from 500 rpm to rest in 10 sec.
5. A body of mass 2kg is fixed to a string 1.5 m long and revolved in horizontal circle about the other end. If it performs 300 rpm calculate its linear velocity, centripetal acceleration and force acting on the body.
6. Racing track of radius of curvature 30 m. It is banked at 30°. Find the maximum speed with which car can move on its track without skidding.
7. Find the angle of banking of a railway track of radius of curvature 250 m. if the optimum velocity of the train 90 km/hr. Also find elevation of outer track over the inner track if the two tracks are 1.6 m apart.
8. Find the angle of banking of a railway track of radius of curvature 1600 m. If the optimum velocity of the train is 20 m/s, find the elevation of outer rail over the inner if the distance between tracks is 1.8 m.
9. Circumference of track is 1.256km. Find the angle of banking of the track, if the maximum speed at which car can be driven safely along it is 25 m/s.
10. A vehicle is moving along a circular road. Which is inclined to the horizontal at 10°. The maximum velocity with which it can move safely is 36 km/hr. Calculate length of the circular road. (Length of circular road = circumference = $2\pi r$)
11. Simple pendulum is suspended from the roof of a carriage of railway train. At what angle will the pendulum string be inclined to the vertical when train goes round the curve of radius 50 m at

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- 45 km/hr.
12. Find the angle which a bicycle and its rider must make with the vertical when traveling at 18 km/hr along a horizontal curve of radius 10 m.
 13. What is angular speed of a second hand of a clock? If the second hand is 10 cm long. Find linear speed of its tip.
 14. Breaking tension of the string is 80 kgwt. A mass of 1 kg is attached to the string and rotated in horizontal circle on a horizontal surface of radius 2 m. Find the maximum no. of revolution made without breaking.
 15. A body of mass 1 kg is fixed to a string and revolves in horizontal circle of radius 1 m. Calculate maximum no. of revolution per minute (r.p.m.). Breaking tension of the string is 9.86 N.
 16. A 1 kg mass is whirled in a horizontal circle at the end of string 0.5 m long. The other end of string being fixed. The breaking tension is 50 N. Find the greatest speed that can be given to the mass.
 17. A coin is kept on a horizontal rotating disc has its centre at a distance of 0.1 m from the axis of rotation of the disc. If coefficient of friction between coin and the disc is 0.25. Find the speed of the disc at which coin is about to slip off.
 18. A coin just remains on the disc rotating at 120 rpm when kept at a distance of 1.5 cm from the axis of rotation. Find coefficient of friction between coin and the disc.
 19. A spring breaks under a tension of 10 kg weight, if a spring is used to revolve a body of mass 1.2 g in a horizontal circle of radius 50 cm. What is maximum speed with which a body can be revolved?
 20. A bucket containing water is tied to one end of 8 m long string and rotated about another end in vertical circle. Find maximum number of rotation per minute in order that water in the bucket may not spill?
 21. Motor cyclist ride in a vertical circle in a hollow sphere of radius 5 m. Find the maximum angular speed required so that he does not lose the contact with the sphere at highest point. ($g = 9.8 \text{ m/s}^2$)
 22. Flyover bridge is in the form of a circular arc of radius 30 m. Find the limiting speed at which car can cross the bridge without losing the contact with the road at highest point assuming centre of gravity of the car to be 0.5 m above the road.
 23. An electron of mass $9 \times 10^{-34} \text{ kg}$ is revolving in a stable orbit of radius $5.37 \times 10^{-34} \text{ m}$. If electrostatic force of attraction between electron and proton is $2 \times 10^{-8} \text{ N}$, Find the velocity of electron.
 24. Air craft takes a turn along a circular path of radius 1500 m. If the linear speed of the air craft is 300 m/s. Find the angular speed and time taken by it to complete $1/5^{\text{th}}$ of circular path.
 25. How fast should the earth rotate so that apparent weight of a body on the equator zero? How long could a day be then? Radius of earth = $6400 \times 10^3 \text{ m}$. $g = 9.8 \text{ m/s}^2$.
 26. Electric fan rotates at 600 rpm is speeded up to 1200 rpm in four sec. Calculate angular acceleration and no. of revolution it has made in this time.
 27. The speed of rotation of disc is reduced from 600 rpm to 300 rpm in 10 sec. Find the no. of rotation made by the disc during this time.
 28. Distance between 2 rails is 1.6 m along a curve of radius 800 m. The outer rail is raised above the inner by 10 cm. With what maximum speed can train be safely driven along the curve?
 29. A train of mass 10^5 kg moves around a curve track of radius 150 m at a speed 20 m/s. find the horizontal thrust on the outer rail if track is not banked. At what angle there is no thrust on the road? Horizontal thrust = Centripetal force.
 30. What is angular speed of a minute hand? If minute hand is 7 cm long. What is its linear speed at its tip?
 31. On end of a string of length 1.5 m is tied to the stone of mass 0.4 kg and the other end to a small pivot on a smooth vertical board. What is the minimum speed of the stone required at its lowermost point so that the string does not slack at any point in its motion along the vertical circle?
 32. Find the minimum speed of a bike at the highest point of a globe of radius 9.8 m.

33. A body of mass 0.5 kg is whirled in a vertical circle by a string 1 m long. Calculate (i) minimum speed it must have at the bottom of the circle so that the string may not slack when the body reaches the top, (ii) in that case, will the velocity at the top of the circle be zero?
34. A ball of mass 10 g is suspended by a string 40 cm long. Keeping the string always taut, the ball describes a horizontal circle of radius 10 cm. Find the angular speed.
35. A body weighing 0.4 kg is whirled in a vertical circle making 2 rps. If the radius of the circle is 1.2 m, find the tension in the string at (i) top of the circle (ii) bottom of the circle.
36. A fighter plane is pulling out for a dive at a speed of 900 km/h. Assuming its path to be a vertical circle of radius 2000 m and its mass to be 1600 kg, find the force exerted by the air on it at the lowest point.
37. Sphere of mass 0.1 kg is attached to an inextensible string of length 1.3 m whose upper end is fixed the ceiling. The sphere is made to describe a horizontal circle of radius 0.5 m. Calculate the time period of revolution and tension in the string.
